THE INERTIAL TIME THEORY

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The current phase of accelerating expansion of the universe is the consequence of an *"inertial force"*. If one performs the apropiate change of coordinates and measures time with a *"standard clock"*, one realises that this accelerating expansion is fictitious and can be explained via a new scale factor asociated to the time coordinate. Therefore, the theory gets rid of the cosmological constant problem and the coincidence problem.

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CONTENTS

I. MOTIVATION OF THE METRIC

Dark Energy (DE) is one of the explanations for the current accelerating expansion of the Universe. Nevertheless, DE cannot be detected and could be interpreted as the mechanism of the inertial force responsible of such acceleration. Conventionally, inertial forces can be explained via change of coordinates. The Friedmann-Robertson-Lemaitre-Walker, FRLW, metric describes a homogeneous and isotropic Universe in expansion. Let's consider the time transformation

$$\mathrm{dt} = \varsigma(\Theta)\mathrm{d}\Theta \tag{1}$$

where t is the cosmic time and Θ is the *"inertial time"* measured by a *"standard clock"*. Thus, the metric reads

$$\mathrm{ds}^2 = \varsigma(\Theta)^2\mathrm{d}\Theta^2 - \mathrm{a}(\Theta)^2\mathrm{d}\mathbf{x}^2 \tag{2}$$

where $a(\Theta)$ is the scale factor asociated to the spatial coordinates and $\varsigma(\Theta)$ is the one asociated to time. Moreover, two Hubble parameters must be defined, namely, the spatial Hubble parameter, $H_a(\Theta) = \frac{\dot{a}}{a}$, and the time Hubble parameter, $H_\Theta(\Theta) = \frac{\dot{\varsigma}}{\varsigma}$, where $\dot{} \equiv \mathrm{d}/\mathrm{d}\Theta$.

II. NEW FRIEDMANN EQUATIONS

One can compute the Einstein tensor, $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$, of the metric (2):

$$G_{00} = 3\frac{1}{\varsigma(\Theta)^2}\frac{\dot{a}(\Theta)^2}{a(\Theta)^2}, \tag{3}$$

$$G_{ij} = \frac{1}{\varsigma(\Theta)^2}\left[2\frac{\ddot{a}(\Theta)}{a(\Theta)} + \frac{\dot{a}(\Theta)}{a(\Theta)}\left(\frac{\dot{a}(\Theta)}{a(\Theta)} - 2\frac{\dot{\varsigma}(\Theta)}{\varsigma(\Theta)}\right)\right]\delta_{ij}. \tag{4}$$

Let's consider a perfect fluid behavior, then the stress energy tensor in units of c is given by $T_{\mu\nu} = (\rho + P)u_\mu u_\nu - g_{\mu\nu}P$

$$T_{00} = (\rho + P)\varsigma^2 - P, \tag{5}$$

$$T_{ij} = -P\delta_{ij}. \tag{6}$$

Therefore, one can compute the Einstein equations, $G_{\mu\nu} = 8\pi G T_{\mu\nu}$. The new Friedmann equations for a Universe with matter and radiation:

$$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho_m\varsigma^4 + \frac{8\pi G}{3}\rho_\gamma\frac{\varsigma^2}{3}[4\varsigma^2 - 1]}, \tag{7}$$

$$\boxed{\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 - \frac{\dot{\varsigma}}{\varsigma}\frac{\dot{a}}{a} = -4\pi G\varsigma^2 P}. \tag{8}$$

Remark if one imposes $\varsigma(\Theta) \equiv 1$, the standard Friedmann equations are recovered.

III. THE FLUID EQUATION

In order to derive the fluid equation [1], one can start from the first law of Thermodynamics:

$$\mathrm{dE} = \mathrm{TdS} - \mathrm{PdV} \tag{9}$$

where $V = \frac{4\pi a(\Theta)^3}{3}$ is the expanding volume in unit of comoving radius, $a(\Theta)$. The change of the energy with the standard time

$$\frac{\mathrm{dE}}{\mathrm{d}\Theta} = T\frac{\mathrm{dS}}{\mathrm{d}\Theta} - P\frac{\mathrm{dV}}{\mathrm{d}\Theta} \tag{10}$$

the change of the volume is computed and adiabiatic expansion is asumed.

The energy $E = mc^2$ and one can write the mass as $m = V\rho$, then

$$\frac{\mathrm{dE}}{\mathrm{d}\Theta} = 4\pi a(\Theta)^2 \rho(\Theta) c^2 \frac{\mathrm{da}}{\mathrm{d}\Theta} + \frac{4\pi}{3} a(\Theta)^3 c^2 \frac{\mathrm{d}\rho}{\mathrm{d}\Theta}. \tag{11}$$

Therefore

$$\boxed{\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + P/c^2) = 0}. \tag{12}$$

If one considers barotropic fluid, i.e. $P = \omega\rho$ ($\omega_{matter} = 0$ and $\omega_{rad} = 1/3$), the solution for matter is

$$\rho_m(\Theta) = \frac{\rho_{m,0}}{a(\Theta)^3} \tag{13}$$

and for radiation

$$\rho_\gamma(\Theta) = \frac{\rho_{\gamma,0}}{a(\Theta)^4}. \tag{14}$$

One could now derive the acceleration equation by differenciating the first Friedmann equation (7) with respect to the inertial time and plugging the fluid equation (12)

$$\frac{\ddot{a}}{a} = \frac{8\pi G}{3}\varsigma^2 \rho \left[\left(\frac{\dot{\varsigma}}{\varsigma}\frac{a}{\dot{a}} + 1 + \frac{1}{2}\frac{a}{\dot{a}} \right) \left(-\omega + (1+\omega)\varsigma^2 \right) + \frac{1}{2}\frac{a}{\dot{a}} \left(-\omega + 2(1+\omega)\varsigma\dot{\varsigma} \right) \right], \tag{15}$$

where ω is the equation of state parameter.

IV. REDSHIFT

In this section, the new redshift relation is derived from the light propagation [1]. Let's consider the emission of photons from one galaxy and another galaxy which observes them. The equation of a propagating photon is given by $\mathrm{ds} = 0$. With no loss of generality, one can consider a radial light ray. Then, writting the metric (2) in comoving coordinates, it yields

$$\frac{\varsigma(\Theta)}{a(\Theta)}\mathrm{d}\Theta \equiv \mathrm{d}\chi. \tag{16}$$

To find the total time the ray takes to get from $\chi = 0$ to $\chi = \chi_0$ it is necessary to integrate the equation above

$$\int_{\Theta_{em}}^{\Theta_{obs}} \frac{\varsigma(\Theta)}{a(\Theta)}\mathrm{d}\Theta = \int_0^{\chi_0} \mathrm{d}\chi. \tag{17}$$

Let's consider a light ray emitted a short time interval after, i.e. $\Theta_{em} + \mathrm{d}\Theta_{\mathrm{em}}$. The galaxies are still at the same coordinates, so the observation time $\Theta_{obs} + \mathrm{d}\Theta_{\mathrm{obs}}$. From a similar integral:

$$\int_{\Theta_{em}+\mathrm{d}\Theta_{\mathrm{em}}}^{\Theta_{obs}+\mathrm{d}\Theta_{\mathrm{obs}}} \frac{\varsigma(\Theta)}{a(\Theta)}\mathrm{d}\Theta = \int_0^{\chi_0} \mathrm{d}\chi. \tag{18}$$

The two expression above are equal, therefore

$$\int_{\Theta_{em}}^{\Theta_{obs}} \frac{\varsigma(\Theta)}{a(\Theta)}\mathrm{d}\Theta = \int_{\Theta_{\mathrm{em}}+\mathrm{d}\Theta_{\mathrm{em}}}^{\Theta_{\mathrm{obs}}+\mathrm{d}\Theta_{\mathrm{obs}}} \frac{\varsigma(\Theta)}{\mathrm{a}(\Theta)}\mathrm{d}\Theta, \tag{19}$$

rearranging the integral limits,

$$\int_{\Theta_{obs}}^{\Theta_{obs}+\mathrm{d}\Theta_{\mathrm{obs}}} \frac{\varsigma(\Theta)}{a(\Theta)}\mathrm{d}\Theta = \int_{\Theta_{\mathrm{em}}}^{\Theta_{\mathrm{em}}+\mathrm{d}\Theta_{\mathrm{em}}} \frac{\varsigma(\Theta)}{\mathrm{a}(\Theta)}\mathrm{d}\Theta, \tag{20}$$

Then,

$$\frac{\mathrm{d}\Theta_{\mathrm{obs}}}{\mathrm{d}\Theta_{\mathrm{em}}} = \frac{\varsigma(\Theta_{em})}{\varsigma(\Theta_{obs})}\frac{a(\Theta_{obs})}{a(\Theta_{em})} \equiv \frac{\lambda_{obs}}{\lambda_{em}}. \tag{21}$$

Moreover, the definition of the cosmological redshift is given by $z+1 = \frac{\lambda_{obs}}{\lambda_{em}}$, where λ_{obs} and λ_{em} are the observed and emitted wavelength of the light ray, respectively. Then,

$$\frac{\varsigma(\Theta_{obs})}{\varsigma(\Theta_{em})}\frac{a(\Theta_{em})}{a(\Theta_{obs})} = \frac{1}{1+z}. \tag{22}$$

At the present time, $z=0$, $\frac{\varsigma_0}{\varsigma(\Theta_0)}\frac{a(\Theta_0)}{a_0} = 1$, so one can set both scale factors to be one at the present time, i.e. $\varsigma_0 \equiv 1$ and $a_0 \equiv 1$. Hence,

$$\boxed{\frac{a(\Theta)}{\varsigma(\Theta)} = \frac{1}{1+z}}. \tag{23}$$

V. DISTANCES

- **The comoving angular distance:** one can rewrite the metric in terms of comoving spatial coordinates,

$$\mathrm{ds}^2 = \varsigma(\Theta)^2\mathrm{d}\Theta^2 - \mathrm{a}(\Theta)^2[\mathrm{d}\chi^2 + \chi^2\mathrm{d}\Omega^2], \tag{24}$$

The equation for photons following a radial trajectory, $\mathrm{ds}^2 = 0$, then the comoving angular distance is give by

$$\chi = \int \frac{\varsigma(\Theta)}{a(\Theta)\mathrm{d}\Theta}. \tag{25}$$

- **The Luminosity distance:** is related to the flux, F, and luminosity, L, through the definition

$$F_{obs} = \frac{L}{4\pi d_L^2}. \tag{26}$$

The flux is the energy that passes per unit time through a unit area. At a comoving distance χ_s from a source, its photons are spread out over the comoving surface of area $4\pi s^2$. The luminosity is the total power emitted by a source at all wavelengths. Moreover, the total received power is not the same as the emitted power, namely

$$P_{em} = \frac{h\nu_{em}}{\delta\Theta_{em}} \tag{27}$$

and

$$P_{obs} = \frac{h\nu_{obs}}{\delta\Theta_{obs}} = \frac{P_{em}}{(1+z)^2}, \tag{28}$$

where the redshift relation has been used. Therefore, plugging $L \equiv P_{obs}$ in the definition of the observed flux, one can get

$$d_L = (1+z)\chi_s. \tag{29}$$

VI. TEMPERATURE REDSHIFT RELATION

The aim is to obtain the temperature-redshift relation [2]. For that purpose, it is necessary to write the particle number density, n, and to make use of the Gibbs law of Thermodynamics and the fluid equation (12) for radiation. The former one can be written as

$$\dot{n} + 3\frac{\dot{a}}{a}n = \psi, \tag{30}$$

where ψ is the particle source term. And the Gibbs law is given by

$$nT\mathrm{d}\sigma = \frac{\rho + \mathrm{P}}{\mathrm{n}}\mathrm{dn} \tag{31}$$

where σ is the specific entropy and T the temperature. Since $\mathrm{d}\sigma$ is an exact differential,

$$T\left(\frac{\partial P}{\partial T}\right)_n = \rho + P - n\left(\frac{\partial \rho}{\partial n}\right)_T. \tag{32}$$

The fact that T and n are thermodynamic independent variables implies that

$$\frac{\dot{T}}{T} = \left(\frac{\partial P}{\partial \rho}\right)_n \frac{\dot{n}}{n} - \dot{\sigma}, \tag{33}$$

$$\dot{\sigma} = \frac{\psi}{nT\left(\frac{\partial\rho}{\partial T}\right)_n}[\rho + P]. \tag{34}$$

Since there is no particle source for radiation, $\psi = 0$, the specific entropy is constant and $\left(\frac{\partial P}{\partial\rho}\right)_n = 1/3$. Then,

$$\frac{\dot{T}}{T} = \frac{1}{3}\frac{\dot{n}}{n}. \tag{35}$$

Combining equations (30) and (35), one obtains the temperature-redshift relation

$$\boxed{T(z) = \frac{T_0}{a} = \frac{T_0(1+z)}{\varsigma}}. \tag{36}$$

In the specific case $\varsigma \equiv 1$, the standard relation is recovered.

VII. SUPERNOVAE ANALYSIS

The distance modulus, $\tilde{\mu}(z)$, is the difference between the apparent magnitude, m, and the absolute magnitude, M,

$$\tilde{\mu}(\tau_1) = m - M = 5\log d_L + \tilde{M} \tag{37}$$

and it is directly related to the luminosity distance. $\tilde{M}$ is considered as a nuissance parameter and must minimize the theoretical expression of chi-square.

Analysis in progress.

[1] A. R. Liddle, Chichester, UK: Wiley (1998) 129 p

[2] J. A. S. Lima, A. I. Silva and S. M. Viegas, Mon. Not. Roy. Astron. Soc. **312** (2000) 747.