THE UNIVERSE in a COCOnut-shell

STANDARD THEORY OF COSMOLOGICAL PERTURBATIONS

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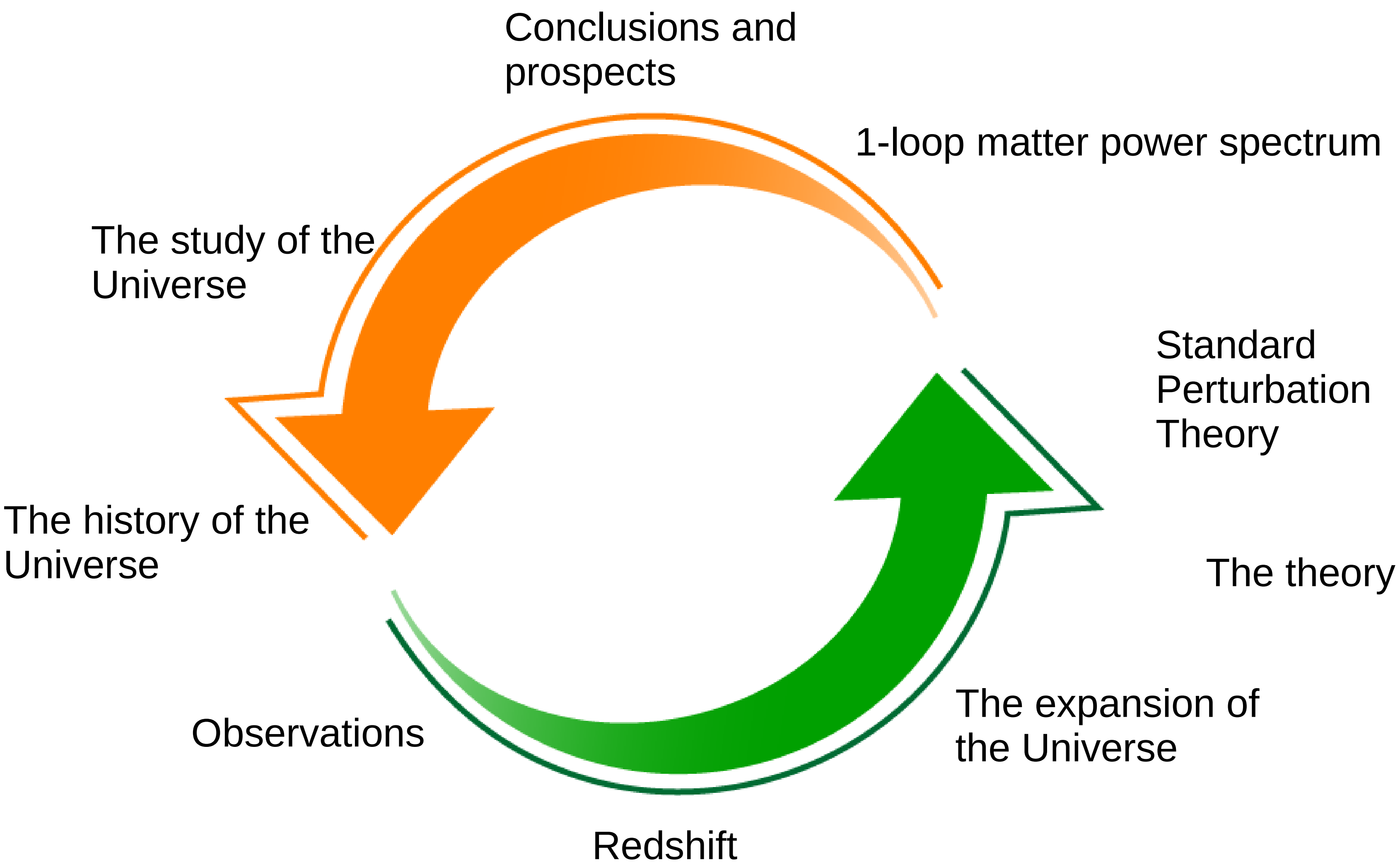
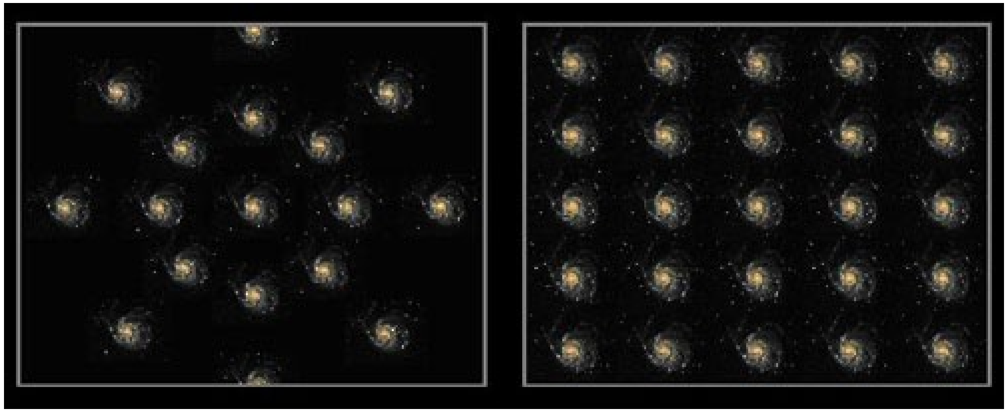
Outline


Conclusions and prospects
1-loop matter power spectrum
Standard Perturbation Theory
The theory
The expansion of the Universe
Redshift
Observations
The history of the Universe
The study of the Universe

The study of the Universe

And the distribution of matter throughout the Universe

Isotropic (rotational invariance)



Homogeneous (translational invariance)

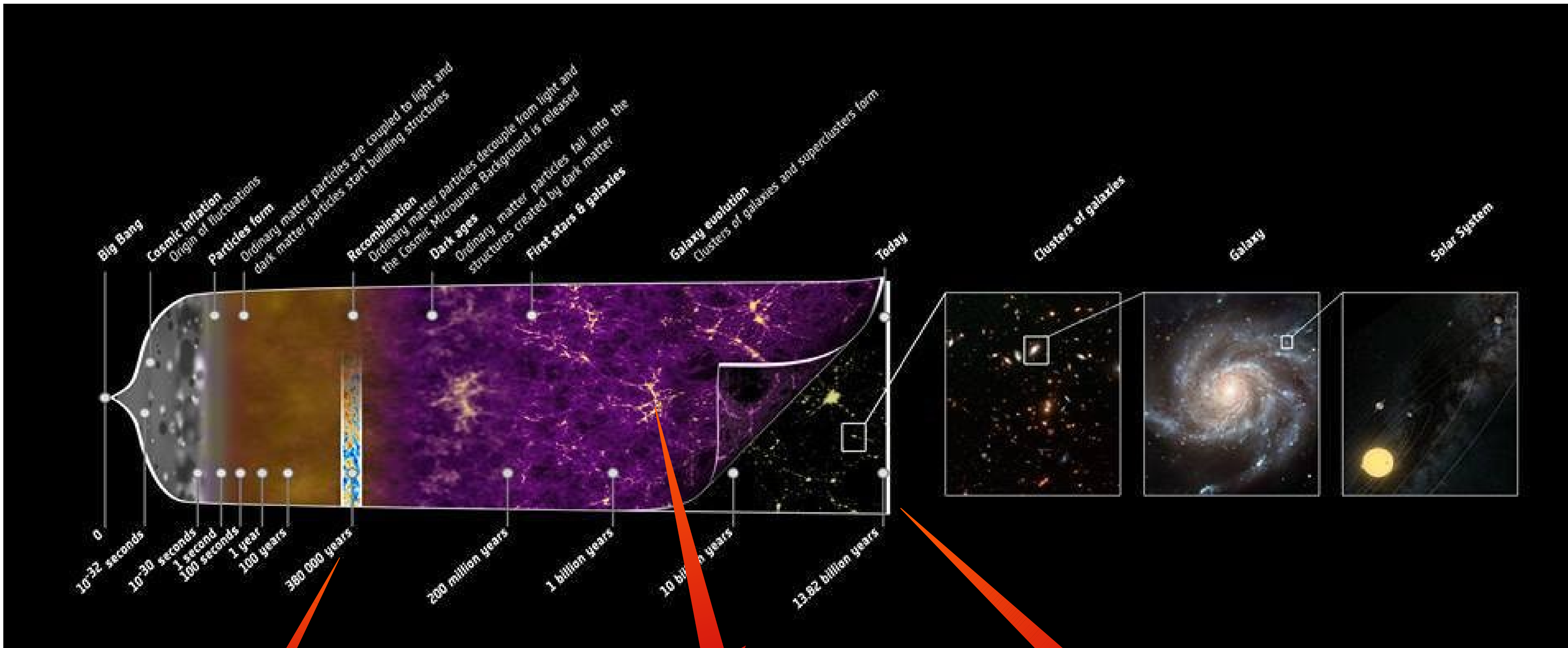
Cosmological principle

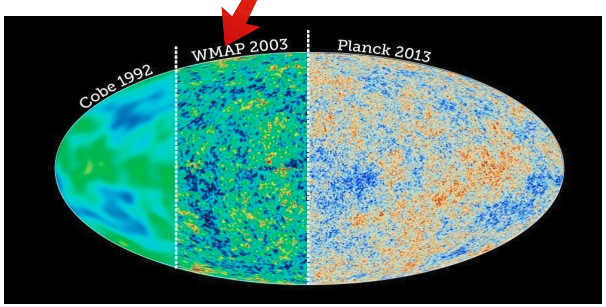
Real Universe

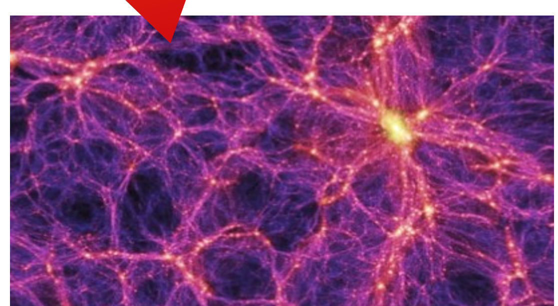
Small asymmetries and fluctuations risen from the beginning of time

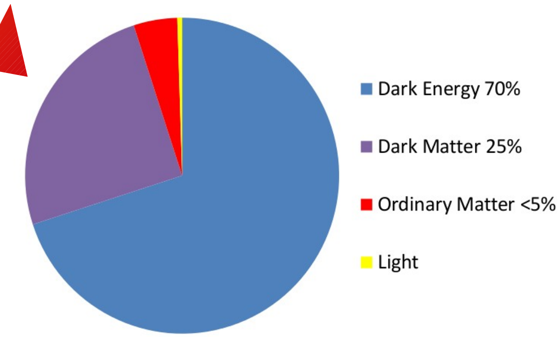
STRUCTURES today!

The history of our Universe



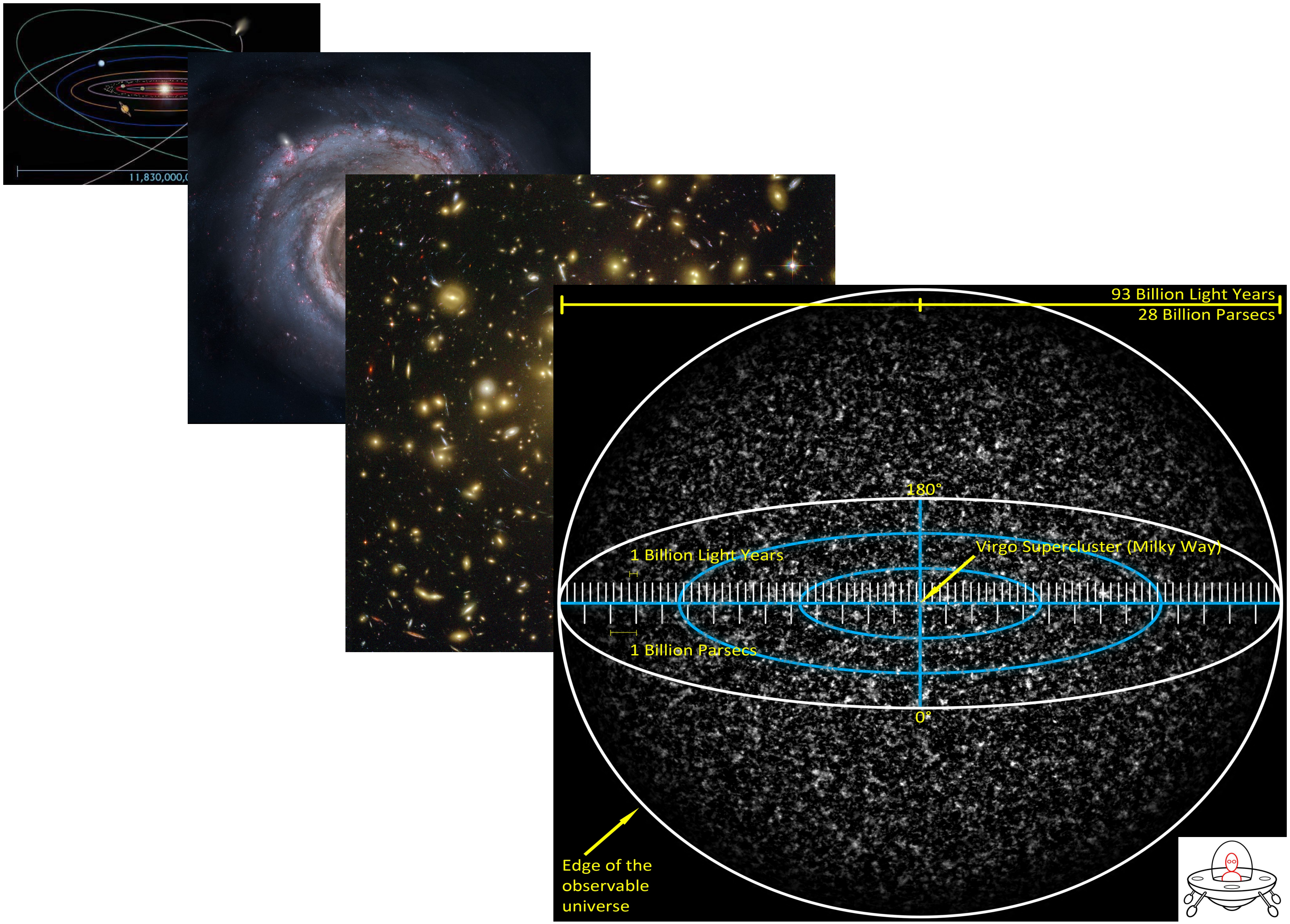






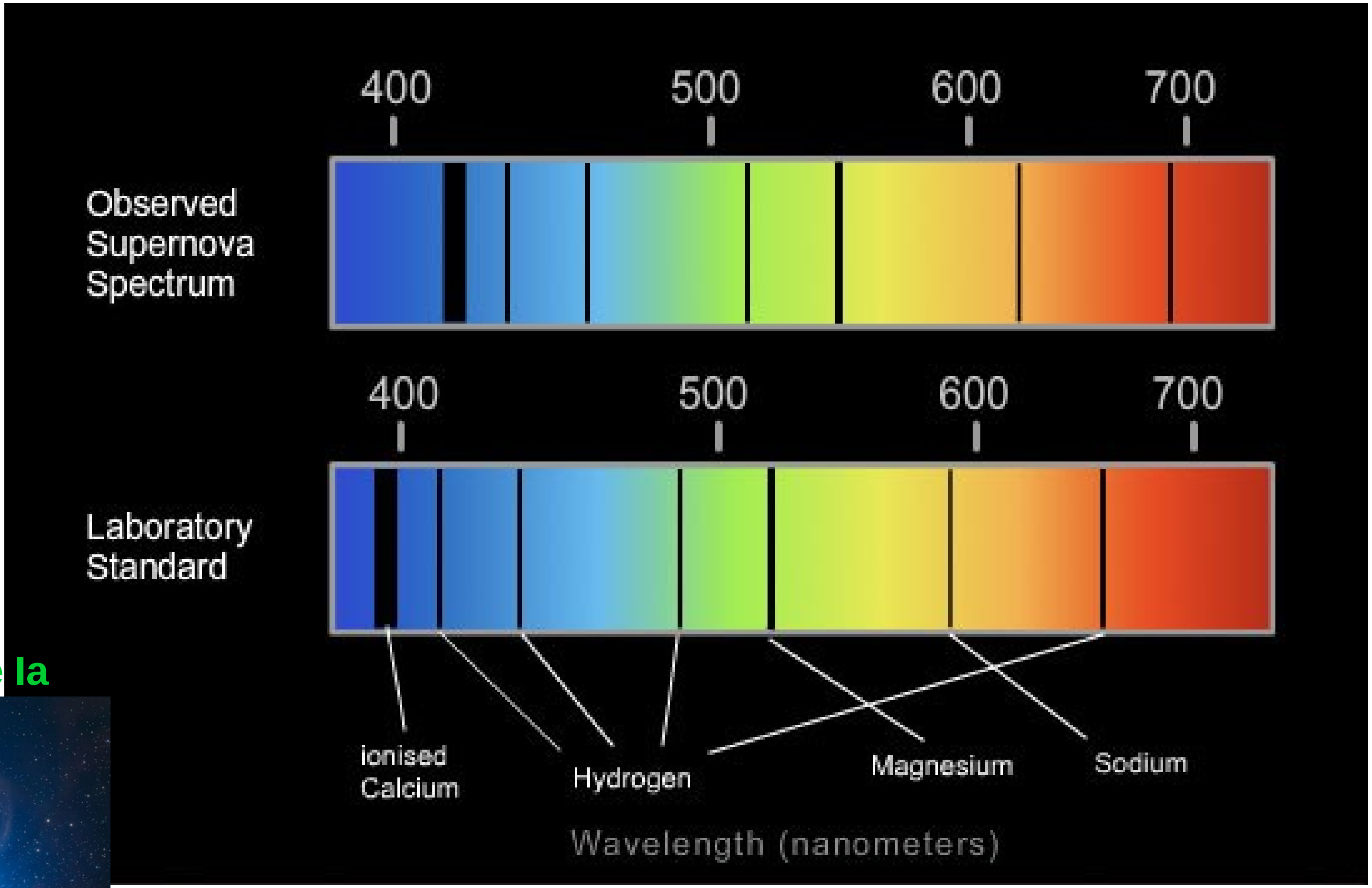
I. OBSERVATIONS

Observations

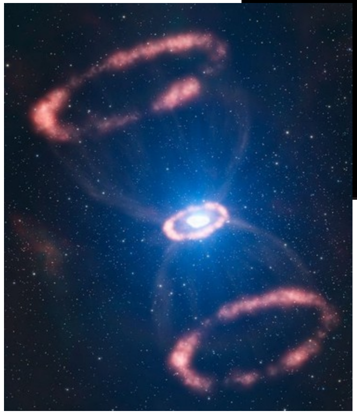


Redshift

$$z = \frac{\lambda_o - \lambda_e}{\lambda_e}$$



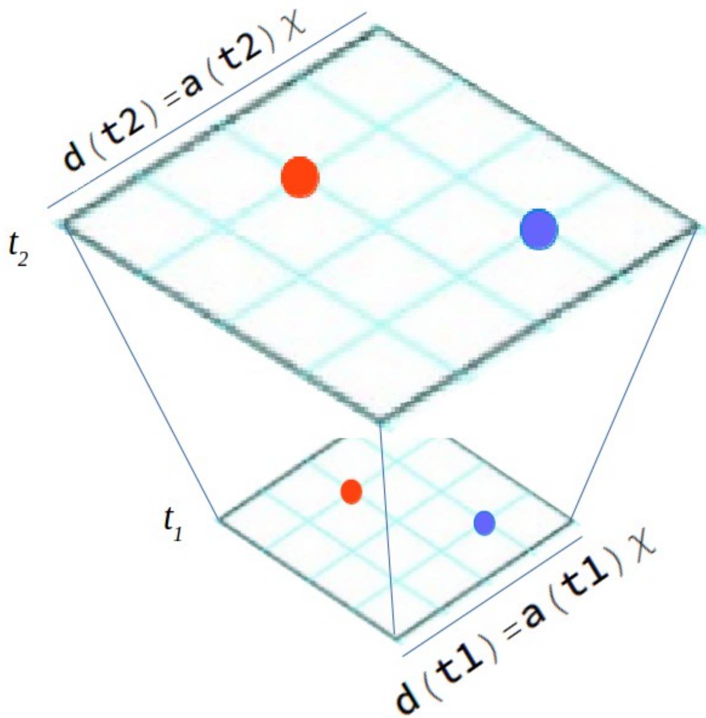
Supernovae Ia



Redshift → Distances

→ Time *Z=0 means "present time"*

The expansion of the Universe



<u>Fundamental observers</u>:
their coordinates do not change with the expansion of the universe.

1. Co-moving coordinate system (grid)
Co-moving distance, χ.

2. Physical distances, *d(t)*, change with the **scale factor, *a(t)*,** as the universe expands.

Expansion rate or **Hubble parameter:** $H(t)=\frac{\dot{a}}{a}$

Redshift – scale factor $a(t)=\frac{1}{1+z}$

Hubble's law

Hubble flow (recession) **velocity:** $\frac{\partial d(t)}{\partial t}\equiv \upsilon = H\,d(t)$

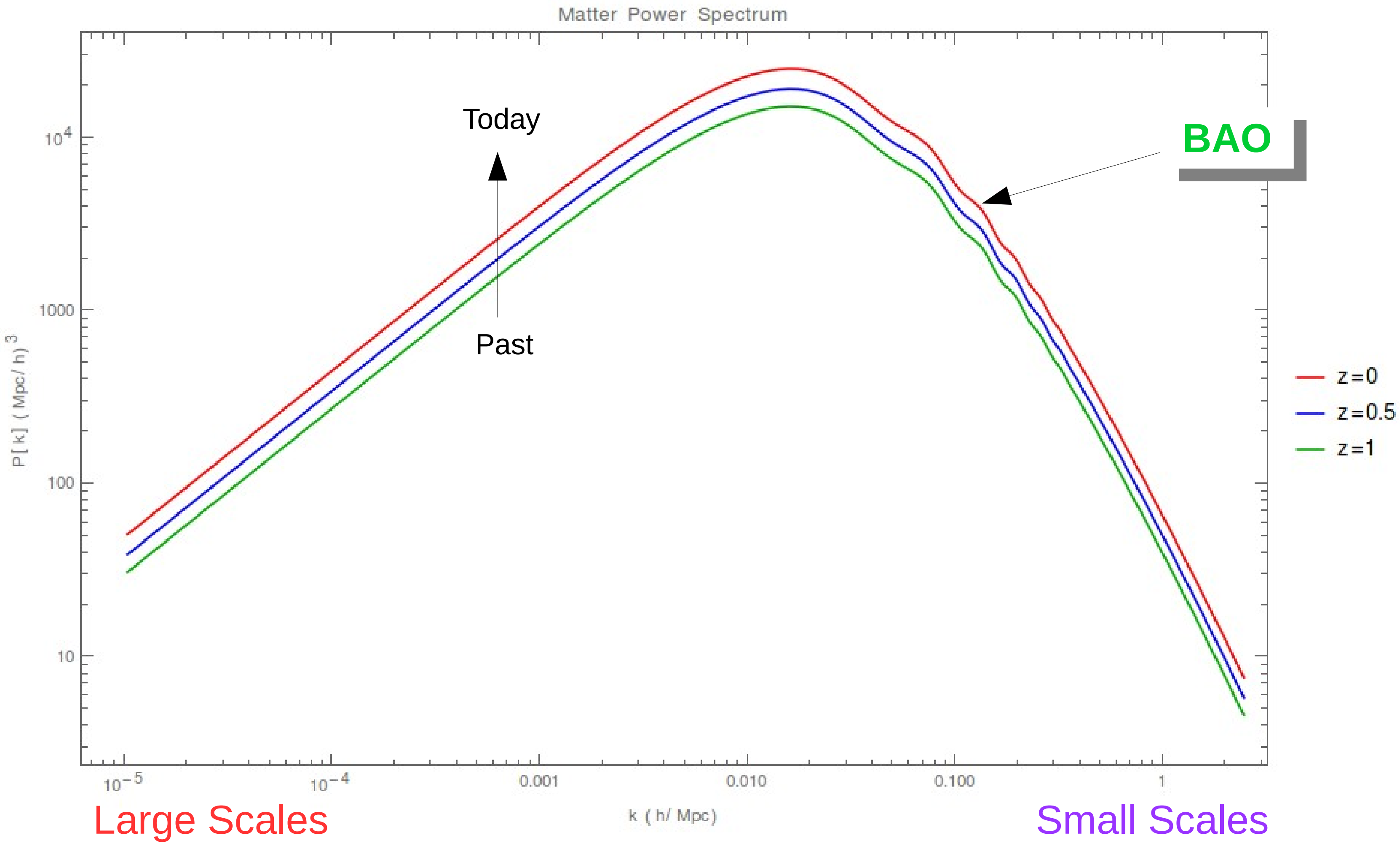
Relation redshift - distances

Peculiar velocity: (anisotropy) $\upsilon_{pec}=a(t)\frac{\partial \chi}{\partial t} \;\rightarrow\; \upsilon = H\,d(t)+\upsilon_{pec}$

Power Spectrum

WHAT TO DO WITH ALL THIS INFORMATION?

Power Spectrum



Planck 2015 cosmology

CAMB [A. Lewis]

II. STANDARD PERTURBATION THEORY

Description of observations

General Relativity
How to measure distances?

Flat, homogeneous and isotropic universe

$$ds^2 = -e^{2\Psi} dt^2 + a^2 e^{2\Phi} dx^2$$

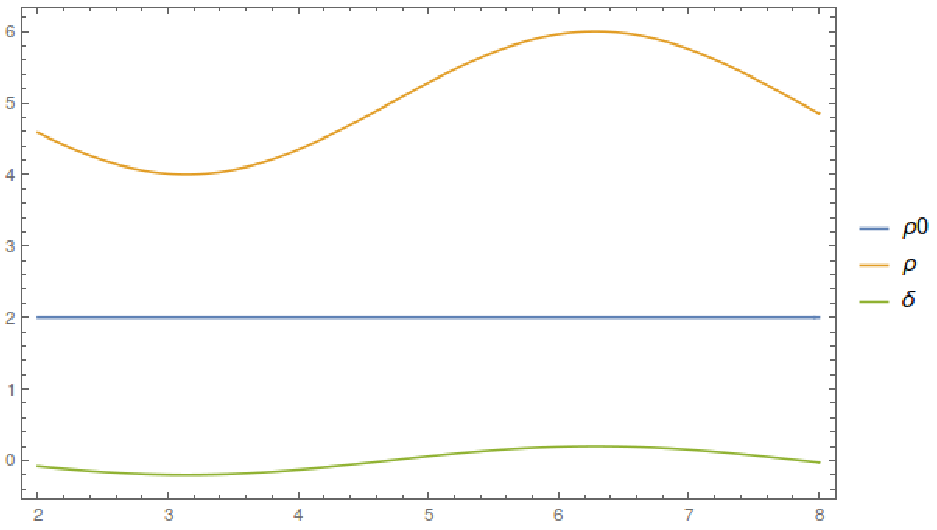
Variational principle

Fluid equations + **Poisson constraint**
energy density ρ and pressure P

Dark Matter (non relativistic)

$$P = 0$$
$$\rho = \rho_0 + \delta\rho$$
$$\delta = \frac{\delta\rho}{\rho}$$

Density contrast



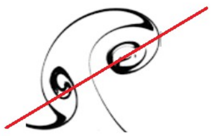
Fluid equations

Density conservation	$\dot{\delta} + \underline{\nabla} \cdot ((1+\delta)\underline{v}) = 0$ [1]
Newton's law	$\dot{\underline{v}} + (\underline{v} \cdot \underline{\nabla})\underline{v} + 2H\underline{v} - \frac{1}{a^2}\underline{\nabla}\Phi = 0$ [2]
Poisson's equation	$\frac{1}{a^2}\nabla^2\Phi = -\frac{3H^2}{2}\Omega_m \delta$ [3]

Cooking up:

$$\Omega_m = \rho_m / 3H^2 M_p^2$$

1. Re-arrange terms to get a single second-order equation for delta.
2. Fourier space.
3. Time derivatives $\dot{\square} \equiv \frac{\partial}{\partial t}$ → redshift derivatives $\square' \equiv \frac{\partial}{\partial z} = -\frac{H}{a}\frac{\partial}{\partial t}$
4. Assumptions Perfect fluid behaviour, non-relativistic limit, radial inflow and negligible vorticity.



The matter density equation

$$\epsilon = -\frac{\dot{H}}{H^2}$$

$$\delta_k'' - \frac{1-\epsilon}{1+z}\delta_k' - \frac{3}{2}\frac{\Omega_m}{(1+z)^2}\delta_k =$$

Linear equation

Non-linear contribution

$$-\int \frac{d^3\underline{q}\, d^3\underline{s}}{(2\pi)^6}(2\pi)^3\delta(\underline{k}-\underline{q}-\underline{s})S_2(\underline{q},\underline{s})$$
$$-\int \frac{d^3\underline{q}\, d^3\underline{s}}{(2\pi)^6}(2\pi)^3\delta(\underline{k}-\underline{q}-\underline{s})\int \frac{d^3\underline{\tau}\, d^3\underline{\sigma}}{(2\pi)^6}(2\pi)^3\delta(\underline{s}-\underline{\tau}-\underline{\sigma})S_3(\underline{q},\underline{s},\underline{\tau},\underline{\sigma})$$
$$+\,O(\delta^4)$$

[4]

Green's method

Solving the equation... via expansion in delta $O(\delta^3)$ (~1-loop)

Linear solution $\delta^{(1)}(k,z) = D(z)\tilde{\delta}_k$

Initial conditions in the past within **matter-dominated era**

$\tilde{\delta}_k = \delta_k(\tilde{z}=50)$

$$D'' - \frac{1-\epsilon}{1+z}D' - \frac{3}{2}\frac{\Omega_m}{(1+z)^2}D = 0$$

Growth function

Second-order

Third-order solution

Insert	in	To get
$\delta_k^{(1)}$	$S_2(\underline{q},\underline{s})$	$\delta^{(2)}(k,z)$
$\delta_k^{(1)}, \delta_k^{(2)}$	$S_3(\underline{q},\underline{s},\underline{\tau},\underline{\sigma})$	$\delta^{(3)}(k,z)$

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)}$$

2-Point Correlation Function

$$\langle \delta^{(n)}(k_1, z)\,\delta^{(n)}(k_2, z)\rangle = (2\pi)^3 \delta(\underline{k_1} + \underline{k_2}) P_{nn}(k, z)$$

Tree level

$$P_{11}(k,z) = D(z)^2 \tilde{P}(k) \qquad [5]$$

1 loop corrections

$$P_{22}(k,z) = D_A^2(z) P_{AA}(k,z) + D_A(z) D_B(z) P_{AB}(k,z) + D_B^2(z) P_{BB}(k,z) \qquad [6]$$

$$\begin{aligned} P_{13}(k,z) = {} & D(z)\tilde{P}(k)[(D_D(z) - D_J(z)) P_D(k,z) + D_E(z) P_E(k,z)] \\ & + D(z)\tilde{P}(k)[(D_F(z) + D_J(z)) P_F(k,z) + D_G(z) P_G(k,z)] \\ & + D(z)\tilde{P}(k)[\frac{D_J(z)}{2}(P_{J2}(k,z) - 2P_{J1}(k,z))] \quad . \end{aligned} \qquad [7]$$

$\tilde{P}(k) \equiv P(k, \tilde{z} = 50)$

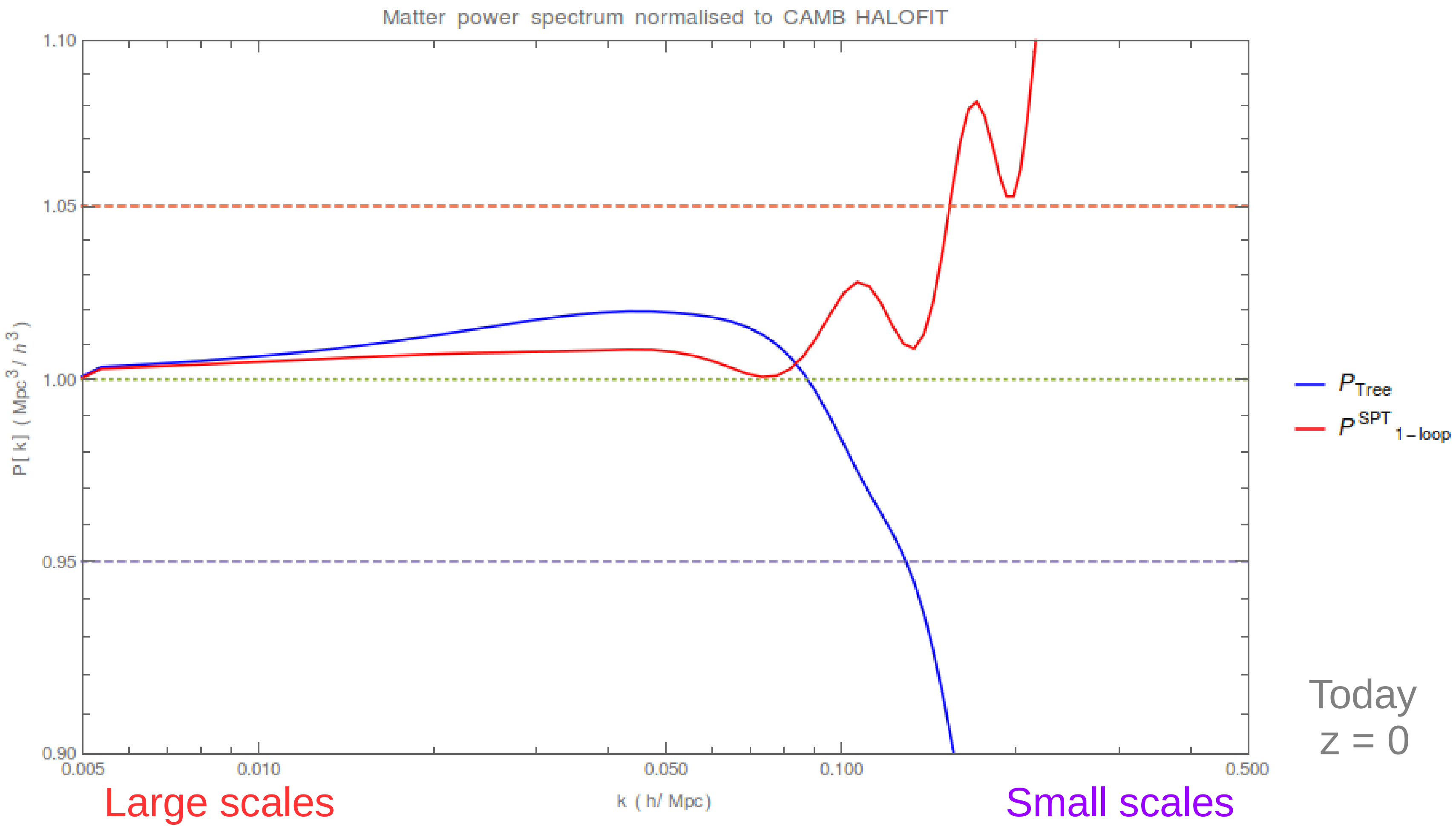
Initial power spectrum by CAMB

$D_{IJ}\ \forall\, I, J = A, B$ AND $D_l\ \forall\, l = D, E, F, G, J1, J2$ BEING GROWTH FUNCTIONS

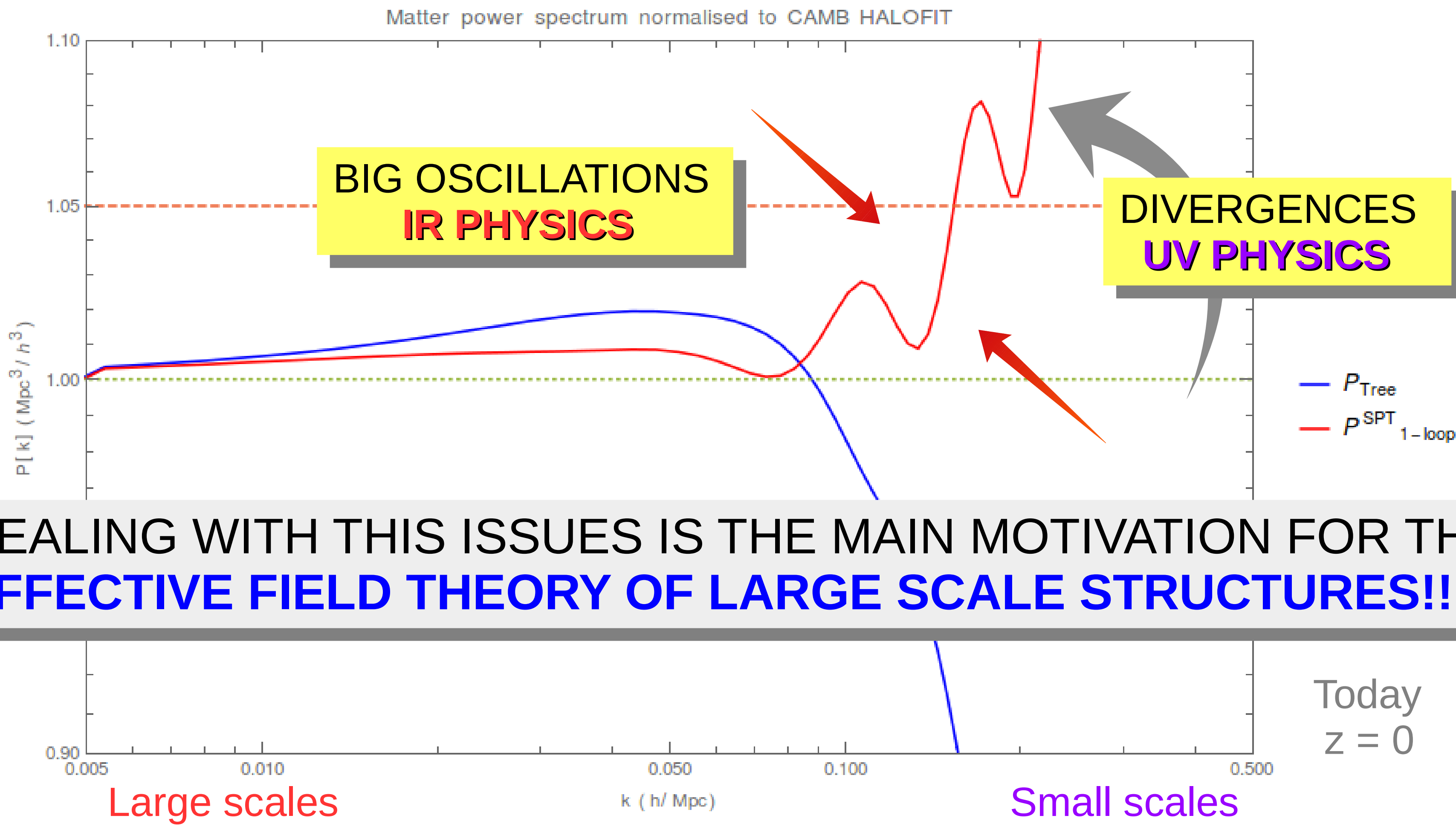
$P_{IJ}\ \forall\, I, J = A, B$ AND $P_l\ \forall\, l = D, E, F, G, J1, J2$ BEING LOOP INTEGRALS

$$P^{SPT}_{1-loop}(k,z) = P_{11}(k,z) + P_{13}(k,z) + P_{22}(k,z) \qquad [8]$$

1-loop matter power spectrum



1-loop matter power spectrum



DEALING WITH THIS ISSUES IS THE MAIN MOTIVATION FOR THE
EFFECTIVE FIELD THEORY OF LARGE SCALE STRUCTURES!!!

Conclusions and Prospects

- The 1-loop matter power spectrum involves weighted integrals over the initial power spectrum with certain weighting functions.
- Similarly to Quantum Field Theory, the influence of the amplitude of fluctuations can be very large or even unbounded in the ultra-violet regime.
- Unreliable predictions: ignorance of an ultimate high-energy theory and troubles to accurately model details of non-linear halo, galaxy formation, gas dynamics...
- We need a re-normalization procedure to account for the effects of ultra-violet physics.
- IR-resumation?
- What about Redshift Space Distortions?