On large-scale structure and cosmological constraints for quintessence and Brans-Dicke models

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In the context of quintessence and Brans-Dicke models we confront the predictions for the expansion history and the matter growth rate with updated measurements of the luminosity distance from supernovae type Ia, angular distance from CMB and galaxy power spectra from several datasets. The use of measurements not only sensitive to the expansion history but to the growth of perturbations enables to break the usual degeneracy of such models with the cosmological Concordance ΛCDM model. Using the Effective Field Theory formalism for Dark Energy, we obtain the confidence regions from the three datasets in the parameters planes and compare the models predictions with dark energy models with constant equation of state.

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I. INTRODUCTION

The epistemological and observational limitations faced by the cosmological concordance ΛCDM model have led the scientific community to suggest a plethora of alternative theories. Aiming to account for the current accelerated phase experienced by the Universe, one broad line of reasoning consists of using gravitational actions beyond the standard Einstein-Hilbert plus a cosmological constant. In this sense, a popular line of reasoning consists of modifying the concordance model avoiding a full modification of the field equations and thus the resulting theories remain inside the Einstein gravity framework. The price to pay consists of assuming the presence of a new component dubbed dark energy (DE) [1], where a possible time (or equivalently redshift) evolution in its energy density is encoded in the equation of state. This is the case of the ωCDM where the dark energy equation of state is constant but different from -1, scalar fields with non-canonical kinetic terms (k-essence) [2], Chaplygin gas model [3] and extra dimensional theories [4] among others. One of the most popular attempts herein has been the so-called Quintessence models [5] where a minimally coupled evolving scalar field plays the role of the dark energy component. The minimal coupling in Quintessence models [6, 7] makes them attractive since the interpretation of the scalar field as resulting from a new cosmological fluid is thus neatly transparent and non-minimal couplings are not required. We shall analyze two Quintessence models: the first will be the so-called Inverse Power-Law model (IPL) [8] characterized by a potential of the form $1/\phi^\alpha$ and a set of three parameters being only two of them independent. The IPL scalar field has attractor-like solution, i.e., for a large range of initial conditions it always converges to the same evolution. This happens in the tracker regime, when the model presents a ωCDM behavior. The second model to be considered is the Double Exponential Potential model (2EP) [9] which exhibits a scaling behavior and whose potential is defined with two exponentials. The 2EP is thence characterized by four parameters, being only three of them independent and two of them also constrained by Nucleosynthesis [9], as it is detailed in following sections.

In addition, a complementary set-up, instead of introducing a new fluid driving the acceleration, consists of obtaining this effect directly from the geometric part of the field equations (*c.f.* [10] for thorough reviews), giving rise to different modified - or extended - gravity theories. In this way DE can be thought of as having a geometrical origin, rather than being attributed to the vacuum energy or additional scalar fields which are added by hand to the stress-energy tensor. Paradigmatic examples of geometrical modifications of the gravitational interaction are the scalar-tensor theories in which a scalar field supplements the standard Einstein-Hilbert Lagrangian [11]. Theoretical arguments supporting the need of scalar-tensor theories rely, among others, in the fact that scalar partners of the graviton naturally arise in most attempts to quantise gravity or unify it with other interactions. Thus, one of the earliest attempts developing an alternative to General Relativity (GR) was done by Brans and Dicke and was indeed in connection with some previous work of Jordan and Fierz [12]. This theory is usually referred to as Jordan-Fierz-Brans-Dicke theory, or also Brans-Dicke theory (BD) (see [13] for a recent review).

In order to constrain the validity of any cosmological theory, studies have usually considered observations

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which are only sensitive to the expansion history, for instance through SNIa, BAO and CMB data. However the use of large-scale observations, such as the aforementioned, which solely depend on the expansion history of the Universe might be not enough to determine uniquely either the nature nor the origin of the mechanism responsible for the late-time acceleration. In other words, cosmological evolutions from different theories can fit the expansion history data with comparable quality, which is usually referred to as the *degeneracy problem*. In order to circumvent this degeneracy, one requires the use of measurements not only sensitive to the cosmological expansion but, for instance, to the growth of structures derived from the evolution of scalar perturbations [15] or other astrophysical and cosmological tests, such as the existence of GR-predicted astrophysical objects, or the evolution of the CMB tensor perturbations [16].

In this investigation we have thus focused our attention in combining constraints coming from SNIa and CMB data with the matter growth rate behavior for Brans-Dicke theories as well as Quintessence-like scalar fields. The theoretical predictions of these theories will be confronted with SNIa [29], CMB [30] and a large variety of dark energy surveys providing galaxy power spectra data, namely THF [17], DNM [18], 6Degree Field Galaxy Survey 6dFGS [19], 2 degree Field Galaxy Red Survey (2dFGRS) [20, 21], 2SLAQ [22], Sloan Digital Sky Survey Luminous Red Galaxies SDSS LRG [23, 24], the Baryon Oscillation Spectroscopic survey BOSS [25], WiggleZ [26] VIMOS-VLT Deep Survey VVDS [21, 27] and VIPERS [28] datasets. From these dark energy surveys, we are interested in the growth structure function, i.e., $f(z)\sigma_{8,0}\delta(z)$, where δ holds for the matter density contrast, $f(z) \equiv \frac{d\ln\delta}{d\ln a}$ represents the growth rate as a function of the redshift, $\sigma_{8,0} \equiv 0.8$ by convention (it splits up the linear from the non-linear regime at scales $8\mathrm{Mpc}\,\mathrm{h}^{-1}$) and $\sigma_{8,0}\delta(z) \equiv \sigma_8(z)$ is the amplitude of the power spectrum of the density perturbations. Previous literature have proposed an approximation to the growth rate in terms of a set of the so-called growth index parameters [31]. Other attempts have tried to encode all the information about the perturbation sector using a single scalar quantity, the so-called growth parameter, which can be constrained with observational data [31]. Also, inspired by the behaviour fourth-order gravity models in the quasi-static limit, the two gravitational potentials were parameterized in terms of a time and scale- dependent Newton's constant and the so-called gravitational slip [32]. Under this assumption numerical codes computing the growth of cosmological perturbations have been implemented, such as MGCAMB [33] and more recently CLASSgal [34] are available depending on the chosen parameterisation. However, in order to represent a useful tool to constrain theories and models therein, the growth parameter must be sufficiently precise. Therefore, solving the linear perturbations equations with no sort of approximation can be more accurate and revealing. This is in fact the strategy to be pursued in this investigation.

In order to perform the calculations on both cosmological background and first order (scalar perturbations) equations we shall use the formalism of Effective Field Theory (EFT) of DE since it provides a well-defined framework with a large classification of theories. It also allows to identify them easily and have the possible instabilities under control. EFT of DE was proposed as a universal description of DE modified gravity theories [7, 35–39] extending a formalism previously applied to inflation [40]. Therein, the metric is considered to be universally coupled to matter fields, hence one can write the most general unitary gauge action compatible with the residual unbroken symmetries of spatial diffeomorphisms. EFT can also be applied to cosmological perturbations by treating them as Goldstone bosons of spontaneously broken time-translations. The operators can be organized in powers of the number of perturbations, hence those beyond the linear order do not affect the background evolution. Besides, the terms in the expansion have direct observable implications. The main advantage of the EFT lies in the transparent manner of classifying large classes of theories by gathering them within the same formalism. This allows us to study not only model-by-model features, but classes of models at once. For instance, one can narrow down theories fulfilling stability conditions for the desired range of parameters values which determine the quoted structural functions and drop out those which does not. Once this is done, one can focus on specific models to be studied in detail. In the following, we shall present the rudiments of this formalism in both background and scalar perturbations evolutions.

This paper is organized as follows: in Section II we present a a brief review of the main features of the EFT formalism for both background and perturbation sectors. Then Section III revises the main properties for the aforementioned Quintessence models as well as for the Brans-Dicke theories, making explicit the evolution and density contrast equations. Section IV is then devoted to computing the luminosity and angular distances and the growth rate of density contrast. Therein we shall perform the corresponding cosmological fits for every model under consideration by making use of χ^2 analyses. Then in Section V we shall present the confidence regions for the pertinent spaces of parameters in the models under study. Finally, in Section VII we present the conclusions of the investigation.

Unless otherwise specified, greek indices run from 0 to 3. The symbol ∇ represents the usual covariant derivative and we use the $(-,+,+,+)$ signature.

II. EFT FORMALISM FOR BACKGROUND AND PERTURBATION SECTORS

This section is devoted to provide a brief review of the theoretical aspects of EFT of DE, namely the introduction of the EFT action, the convenience of working in the unitary gauge and the equations for the relevant sectors. Further details are provided in references [35, 36].

Action

The use of the unitary gauge allows us to express the action only in terms of the metric tensor and its derivatives. This happens since the scalar field is taken as the time coordinate while the spatial coordinates remain unfixed (see [36] for a more exhaustive reading). The low energy spectrum and the dynamics are described in a general way, regardless of higher energy levels of the theory. Besides, the spontaneous breaking of a global symmetry is considered. Thus the Lagrangian is written in the unitary gauge (such that it is invariant under the unbroken symmetries but non-invariant under the broken symmetries). Furthermore, the diffeomorphism invariance is restored by the so-called *Stückelberg mechanism* which is nothing but the imposition of a time coordinate transformation on the action (since the time is fixed by applying the unitary gauge), i.e., $t \rightarrow t + \pi(x^\mu)$ [35, 36].

In a general perturbed RW spacetime, the perturbed scalar field takes the form $\phi(t,\vec{x}) = \phi_0(t) + \delta\phi(t,\vec{x})$, but since $t = t(\phi)$ by applying the unitary gauge $\delta\phi = 0$. Moreover, every $\phi = const$ defines a time slicing, so that one can build the action with the unit vector n_μ orthogonal to such a slicing, regardless of the scalar ϕ,

$$n_\mu \equiv -\frac{\partial_\mu \phi}{\sqrt{-(\partial\phi)^2}} \rightarrow -\frac{\delta^0_\mu}{\sqrt{|g^{00}|}} \tag{1}$$

this means that besides any curvature invariant such as the Ricci scalar R, also contractions of tensors with free upper 0 indices (e.g. g^{00}, R^{00}, etc.) with n_μ are allowed. Furthermore, coefficients which multiply the operators in the action are allowed to be time dependent because time translations are broken. Moreover, covariant derivatives of the unit defined by $\phi = const.$ are used to express the operators. In an equivalent manner, projection orthogonal to $\phi = const.$ or $t = const.$ surfaces can be used, so that the extrinsic curvature becomes

$$K_{\mu\nu} \equiv h^\sigma_\mu \nabla_\sigma n_\nu, \tag{2}$$

where $h_{\mu\nu} \equiv g_{\mu\nu} + n_\mu n_\nu$ is the so-called induced spatial metric.

It is worth noting that EFT of inflation is nothing but the propagation of a scalar degree of freedom on a general RW background for which time translations are unbroken [40]. In order to extend the EFT formalism from inflation to late-time cosmology, i.e., EFT of DE, matter fields must be taken into account (e.g. dark matter, radiation, etc.). Therefore, for the matter sector the *weak equivalence principle* is assumed to be valid, hence matter fields couple to the metric through a covariant action and working within the Jordan frame turns out to be more advantageous.

Theory	$\mu = \frac{\mathrm{d}\log(M^2(t))}{\mathrm{d}t}$	$\lambda(t)$	$C(t)$	$\mu_2^2(t)$	$\mu_3(t)$	$\epsilon_4(t)$
ΛCDM	0	const	0	0	0	0
Quintessence	0	✓	✓	0	0	0
ωCDM	0	✓	0	0	0	0
JFBD	✓	✓	✓	0	0	0

Table I: Parameterizations for ΛCDM, Quintessence, ωCDM and JFBD theories using the Effective field theory formalism. The pragmatic reader should take this table and use it as a recipe, applying it directly to the desired theory or model. A more detailed table can be found in [41, 42].

Thus, using the unitary gauge the dynamics is totally encoded in the degrees of freedom of the metric and the total action then becomes [41, 42],

$$\begin{aligned} S &= \int d^4x\sqrt{|g|}\frac{M^2(t)}{2}\left[R - 2\lambda(t) - 2C(t)g^{00}\right. \\ &+ \mu_2^2(t)\left(\delta g^{00}\right)^2 - \mu_3(t)\delta K\delta g^{00} + \epsilon_4(t)(\mathrm{K}^\mu_{\ \nu}\mathrm{K}^\nu_{\ \mu} - \delta K^2 \\ &+ \left.{}^{(3)}R\,\delta g^{00}/2)\right] + S_m[g_{\mu\nu};\psi], \end{aligned} \tag{3}$$

where $M(t)$, $\lambda(t)$, $C(t)$, $\mu_2^2(t)$, $\mu_3(t)$ and $\epsilon_4(t)$ are the so-called structural functions [41, 42], $\delta g^{00} \equiv 1 + g^{00}$ is the lapse component, $\delta K_{\mu\nu}$ is the perturbation of the extrinsic curvature on hypersurfaces of constant time, its trace is denoted by δK and ${}^{(3)}R$ is the 3-dimensional Ricci scalar on such a hypersurface, whereas $S_m[g_{\mu\nu},\psi]$ refers to the action for the matter fields, ψ. Table I lists some DE and modified gravity theories covered by the above action (3) and it shows how many and which of the structural functions define each theory.

Another relevant aspect of the theoretical development of any extended theory of gravity lies in the stability study. In the EFT of DE, the function $\mu_2^2(t)$ plays the main role in the study of stabilities and the speed of sound of DE. Let us remind that a theory is said to be sound if the propagating scalar degree of freedom has neither ghost instabilities nor gradient instabilities. By forcing the time diffeomorphism $t \rightarrow t + \pi(x)$, on the action, the spacetime dependent parameter $\pi(x)$ becomes the scalar field fluctuation. Once the system is diagonalized with field redefinitions, the actual propagating degree of freedom π, decoupled from gravity, reads as

$$S_\pi = \int a^3M^2\left[A\dot{\pi}^2 - B\frac{(\vec{\nabla}\pi)^2}{a^2}\right] + \mathcal{O}(\pi^2), \tag{4}$$

where

$$A \equiv (C+2\mu_2^2)(1+\epsilon_4)+\frac{3}{4}(\mu-\mu_3), \quad (5)$$

$$\begin{aligned} B &\equiv (C+\tilde{\mu}_3/2-\dot{H}\epsilon_4+H\tilde{\epsilon}_4)(1+\epsilon_4) \\ &- (\mu-\mu_3)\left[\frac{\mu-\mu_3}{4(1+\epsilon_4)}-\mu-\tilde{\epsilon}_4\right]. \quad (6)\end{aligned}$$

The expressions A and B must be separately positive: $A > 0$ implies the absence of ghosts and hence that the soundness of the theory is guaranteed. $B \geqslant 0$ enforces the gradient stability condition. Furthermore, the speed of sound or the propagation speed of DE reads as $c_s^2 = B/A$, which must be a positive number, although just with these conditions there is no guarantee for that the speed of sound be less than the speed of light. However, large values of the structural function μ_2^2 prevents from superluminal propagation.

Relevant sectors

The main advantage of the unitary gauge choice is a neat separation between terms contributing to the background evolution and those affecting the perturbations. The relevant equations for every sector will be introduced below.

With regard to the background, the structural functions governing this sector turn out to be $M^2(t)$, $\lambda(t)$ and $C(t)$. Since the matter fields are essentially constituted by non-relativistic species, a perfect fluid approximation is adopted and the matter pressure is set to be zero. The field equations derived from the action for a spatially flat Universe read as

$$C = \frac{1}{2}(H\mu-\dot{\mu}-\mu^2)+\frac{1}{2M^2}(\rho_{\rm DE}+p_{\rm DE}), \quad (7)$$

$$\lambda = \frac{1}{2}(5H\mu+\dot{\mu}+\mu^2)+\frac{1}{2M^2}(\rho_{\rm DE}-p_{\rm DE}). \quad (8)$$

where $H(t)$ is the Hubble parameter, $a(t)$ is the scale factor, dot holds for derivative with respect to time t, $\mu \equiv \frac{\mathrm{d}(\log M^2(t))}{\mathrm{d}t}$ is the non-minimal coupling function, and $\rho_{\rm DE}$ and $p_{\rm DE}$ are the DE density and pressure respectively. The DE equation of state is given by $\omega(t)$. It is worth noting the fact that the background functions completely determine the expansion history $H(t)$, whereas the converse is not true. In other words, different choices of $M^2(t)$ and $\rho_{\rm DE}$ can lead the same Hubble rate, although this degeneracy can be eventually removed [41].

On the other side, the perturbation sector is focused on the study of the large-scale structures evolution for inhomogeneous distribution of matter in the universe. This can be computed by using linear perturbation theory (see for instance [43]) and it is governed by the structural functions $M^2(t)$, $C(t)$, $\mu_3(t)$ and $\epsilon_4(t)$. Assuming the quasi-static approximation[1], the evolution of density contrast δ in the quasi-static approximation for the general EFT formalism, is given by

$$\ddot{\delta}+2H\dot{\delta}-\frac{3}{2}G_{eff}\Omega_m\delta = 0, \quad (9)$$

where $\Omega_m = \frac{8\pi G}{3H_0^2}\rho_m(z)$ and G_{eff} reads as

$$\begin{aligned} G_{eff} &= \frac{M_{Pl}^2}{M^2(1+\epsilon_4)} \quad (10)\\ &\cdot \frac{C+\frac{\tilde{\mu}_3}{2}-\dot{H}\epsilon_4+H\tilde{\epsilon}_4+(\mu+\tilde{\epsilon}_4)^2+Y_{IR}}{C+\frac{\tilde{\mu}_3}{2}-\dot{H}\epsilon_4+H\tilde{\epsilon}_4+\frac{(\mu+\tilde{\epsilon}_4)(\mu-\mu_3)}{1+\epsilon_4}-\frac{(\mu-\mu_3)^2}{4(1+\epsilon_4)^2}+Y_{IR}}\end{aligned}$$

with $G \equiv \frac{1}{8\pi M_{Pl}^2}$, $\tilde{\mu}_3 \equiv \dot{\mu}_3+\mu\mu_3+H\mu_3$, $\tilde{\epsilon}_4 \equiv \dot{\epsilon}_4+\mu\epsilon_4+H\epsilon_4$, and $Y_{IR}(k,t)$ accounts for infrared corrections. However, IR corrections become important only at wavelengths scales as large as Hubble scale, therefore Y_{IR} can be considered negligible [41].

III. BRANS-DICKE AND QUINTESSENCE THEORIES

Brans-Dicke theories

The action corresponding to JFBD theory without potential in the Jordan frame can be written as

$$S_{\rm BD} = \int \frac{\mathrm{d}^4x}{16\pi G_*}\sqrt{-g}\left[\phi R-\frac{\omega_0}{\phi}\partial^\nu\phi\partial_\nu\phi\right]+S_m[g_{\mu\nu};\psi], \quad (11)$$

where G_* holds for the bare gravitational coupling constant, R the Ricci scalar associated to the metric $g_{\mu\nu}$, $\sqrt{-g}$ the determinant of the metric, S_m the action corresponding to matter fields ψ and the metric, ϕ the scalar field and ω_0 the constant coupling between the scalar field and the metric[2] .

[1] For sub-Hubble scales, time derivatives of involved quantities are usually neglected with respect to the spatial derivatives. Numerous investigations have addressed the validity of the quasi-static approximation within the perturbation theory. One must be aware of the error introduced when this approximation is employed [44, 45].

[2] As mentioned, JFBD theories are included in a more general set of the so-called scalar-tensor theories. These theories allow for ω_0 depending on the scalar field itself and may also include a potential to the scalar field Lagrangian density. In fact the popular $f(R)$ theories of gravity correspond to $\omega_0 = 0$ in its metric version [14].

Using unitary gauge in the previous action (11)

$$S_{\rm BD} = \int {\rm d}^4x\sqrt{|g|}\frac{M^2(t)}{2}\left[R-\omega_0 g^{00}\left(\frac{\dot\phi_0(t)}{\phi_0(t)}\right)^2-\tilde V(t)\right], \tag{12}$$

where we have rewritten the scalar field in terms of the mass factor as $M^2(t)\equiv\frac{\phi_0(t)}{8\pi G_*}$ and the potential as $\tilde V(t)\equiv V(t)/\phi_0(t)$. Note ω_0 can be reparameterized as $\omega_0 = -\frac{3-1/\alpha^2}{2}$ [46] where $\alpha < 3.45\times10^{-3}$ according to [47]. Raychaudhuri and evolution equations for JFBD [48, 49] read as

$$H^2+H\frac{\dot\phi}{\phi}-\frac{\omega_0}{6}\left(\frac{\dot\phi}{\phi}\right)^2-\frac{V(\phi)}{6\phi}=\frac{\rho_m}{3\phi} \tag{13}$$

$$2\dot H+3H^2+\frac{\ddot\phi}{\phi}+2H\frac{\dot\phi}{\phi}+\frac{\omega_0}{2}\left(\frac{\dot\phi}{\phi}\right)^2-\frac{V(\phi)}{\phi}=-\frac{p_m}{\phi} \tag{14}$$

and the equation of motion for the scalar field yields

$$\ddot\phi+3H\dot\phi-8\pi G\rho_{m,0}\alpha^2a^{-3}=0, \tag{15}$$

By identifying terms between (3) and (12) one can rewrite BD theories into the EFT formalism, finding that the three non-zero structural functions are (

$$\mu(t) = \frac{\dot\phi_0(t)}{\phi_0(t)}\ ;\ \lambda(t)=\frac{\tilde V(t)}{2}\ ;\ C(t)=\frac{\omega_0}{2}\left(\frac{\dot\phi_0(t)}{\phi_0(t)}\right)^2. \tag{16}$$

Nevertheless, in our analysis only those JFBD model with null potential are considered for the sake of simplicity. Therefore, the structural function $\lambda(t)$ also vanishes.

The considered initial conditions: $\phi(z=1100)=0$ and $\dot\phi(z=1100)=0$ (as it is suggested in reference [46]). Furthermore, the contribution of the scalar field to the background quantities is quite negligible, $\frac{\phi'^2}{H}\sim 10^{-11}-10^{-17}$. Due to this fact, one can take with no loss of accuracy the expansion history as given by ΛCDM, for the JFBD models under study [46].

Concerning the perturbation sector, the expression for the effective gravitational function (10) yields

$$G_{eff}=\frac{2C+2\mu^2+Y_{IR}}{2C+\frac{3}{2}\mu^2+Y_{IR}}, \tag{17}$$

where it is easy to notice that, for subHubble modes one can assume $Y_{IR}\simeq 0$. Thus one can solve the equation (9) with the same initial conditions as for ΛCDM.

Furthermore, it is worth noting that JFBD models fulfill the stability conditions, in other words, equations (5) and (6) are simultaneously positive. Besides, the square of speed of sound (ratio between (6) and (5)) is equal to one, avoiding superluminal fields.

Quintessence models

The general action of a Quintessence single field minimally coupled to gravity [6] reads as

$$S_\phi=\int {\rm d}^4x\sqrt{|g|}\left[\frac{M^2}{2}R-\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)-V(\phi)\right]. \tag{18}$$

After applying therein the unitary gauge, i.e., fixing the time coordinate as a function of the scalar field, the quintessence action (which only displays metric degrees of freedom) takes the following form:

$$S_\phi=\int {\rm d}^4x\sqrt{|g|}\frac{M^2(t)}{2}\left[R-\frac{2}{M^2(t)}\left(\frac{\dot\phi_0^2(t)}{2}g^{00}-V(t)\right)\right] \tag{19}$$

In the case of Quintessence, the only structural functions which are not null are $C(t)$, $\lambda(t)$ and $M(t)\equiv const.$ which can be identified with the terms on the general EFT of DE action (3). This leads to

$$C(t)=\frac{\dot\phi_0^2}{2M^2(t)}\ ;\ \lambda(t)=\frac{V(t)}{M^2(t)}. \tag{20}$$

Besides, one can write the action in terms of the background quantities by computing the stress-energy tensor. Thus, the density energy and the pressure of the scalar fields read respectively as

$$\rho_\phi=\frac{\dot{\phi_0}^2}{2M^2(t)}+V(\phi)\ ;\ p_\phi=\frac{\dot{\phi_0}^2}{2M^2(t)}-V(\phi). \tag{21}$$

Then, the Friedmann equation and the equation of motion for the field take the form

$$\left(\frac{a'(\tau)}{a(\tau)}\right)^2=\frac{\tilde\phi'^2}{6}+\tilde V(\tilde\phi)+\Omega_{m,0}a^{-3}(\tau), \tag{22}$$

$$\tilde\phi''+3\frac{a'}{a}\tilde\phi'+3\frac{\partial\tilde V(\tilde\phi)}{\partial\tilde\phi}=0, \tag{23}$$

being prime the derivative with respect to the dimensionless time $\tau=H_0t$ (where t is the cosmological time and H_0 is the Hubble parameter at the present), $\tilde\phi=\phi\sqrt{8\pi G}$, $\Omega_{m,0}=\frac{8\pi G}{3H_0^2}\rho_{m,0}$ and $\tilde V(\tilde\phi)$ is the dimensionless potential. Finally for Quintessence, the density contrast equation takes the form (9), where $G_{eff}=1$ according to (10).Moreover, Quintessence theories satisfy the ghost and gradient stability conditions and the square of the speed of sound is exactly one. From now on, we particularize our study to IPL and 2EP models of Quintessence.

Inverse Power-Law model

For Quintessence IPL models the potential reads as

$$V(\phi)=\frac{M^{4+\alpha}}{\phi^\alpha}. \tag{24}$$

One of the main features of this kind of models is the appearance of a tracker attractor. The tracker field is characterized by the fact that it remains subdominant for the most of the history of the Universe and it begins to be relevant at late epoch when it drives the cosmic acceleration. Tracker fields have attractor-like solutions which means nothing but that they rapidly converge to a common cosmic evolutionary track of $\rho_\phi(t)$ and $\omega_\phi(t)$ for a wide range of initial conditions. The equation of state is determined by the dominant component, e.g., in the matter-dominated epoch the equation of state of the tracker field reads as $\omega_\phi = \frac{\alpha\omega_m - 2}{\alpha+2} = \frac{-2}{\alpha+2}$ [50]. Let us remind this is valid only when the Quintessence component is not dominant and its equation of state is a constant so that the model could be thought to behave as a ωCDM model.

We solve the Friedmann equation (22) and the equation of motion for the scalar field (23) where the dimensionless potential is $\tilde{V}(\tilde{\phi}) = A/\tilde{\phi}^\alpha$, being $A = \frac{M^{4+\alpha}(8\pi G)^{1+\alpha/2}}{3H_0^2}$. Initial conditions for the background quantities are $a(0) = 1/1001$, and $\phi(0) = 0.5$, $\phi'(0) = 0$ for the scalar field [50].

The dependency between parameters $\Omega_{m,0}$ and A is encoded in the equation of motion (23). This means that one cannot know *a priori* which value of A corresponds to a given $\Omega_{m,0}$. Due to this fact, we need to perform a first integration with arbitrary values $\{\Omega_{m,0}, A\}$, to implement a time rescaling, since the true values of the aforementioned parameters must satisfy the equalities $H(z=0) = 1$ and $a(z=0) = 1$, in order to compute a second integration which yields the true background and field evolution. Once this subtlety is considered, we are ready to integrate the matter density perturbation equation (9) for $\alpha \in [0, 10]$[3] and $\Omega_{m,0} \in [0, 0.50]$.

Double Exponential Potential model

The second Quintessence model is the 2EPmodel whose potential takes the form:

$$V(\phi) = M^4(e^{\alpha\sqrt{8\pi G}\phi} + e^{\beta\sqrt{8\pi G}\phi}), \tag{25}$$

where M, α and β are the parameters of the model. The corresponding dimensionless quantities would be:

$$\tilde{V}(\tilde{\phi}) = A(e^{\alpha\tilde{\phi}} + e^{\beta\tilde{\phi}}) \tag{26}$$

being $\tilde{\phi} = \phi\sqrt{8\pi G}$ and $A = \frac{M^4(8\pi G)}{3H_0^2}$. The reason of considering a double exponential instead of a single one lies in the fact that for the latter, the yielded equation of state is the same as that of the background fluid and then there is no cosmic acceleration phase. However, the use of a double exponential potential solves this problem. Another interesting feature of of the potential (25) above is its behavior along the cosmological eras: during the matter-dominated epoch, it follows a scaling behavior, whereas it becomes dominant at late times inducing an accelerating phase with an equation of state quite close to -1. Different values of the parameters imply either an early or a late transition from the scaling matter era to the accelerating era. The equations of motion for the Hubble rate (22) and the scalar field (23) are solved in the same manner as for the case of IPL with the same initial conditions and they present the same kind of subtleties, so that a double integration must also be performed. In this case, α cannot take either negative values or higher than 0.8 (since $\omega_\phi < -0.8$ [9]) and β can be fixed for the sake of simplicity to $\beta = 20$ [50] since β must be higher than 5.5 due to Nucleosynthesis constraints, see [9].

IV. DATA FIT ANALYSIS

We focus our study in theories which involve a mechanism responsible of the accelerated expansion at the present epoch and which must be subdominant in the past. A χ^2 test is implemented in order to constrain the parameters of the different models.

In the perturbation sector, we use the Wilkinson Microwave Anisotropy Probe 9 (WMAP9) in combination with THF, DNM, 2 degree Field Galaxy Red Survey 2dFGRS, 2SLAQ, VIMOS-VLT Deep Survey VVDS, Sloan Digital Sky Survey Luminous Red Galaxies SDSS LRG, Wiggle Z, the Baryon Oscillation Spectroscopic survey BOSS, 6Degree Field Galaxy Survey 6dFGS and VIPERS datasets, see Table II. In particular, from the large variety of DE Surveys, we are interested in the growth structure functions, i.e. $f(z)\sigma_{8,0}\delta(z)$, where $f(z) \equiv \frac{d\ln\delta}{d\ln a}$ is the growth rate as a function of the redshift, $\sigma_{8,0}$ is defined as the amplitude of the perturbations at the the present time and at the length scale, $r = 8\mathrm{Mpch}^{-1}$, which splits up the linear from the non-linear regime. Further $\sigma_{8,0}\delta(z) \equiv \sigma_8(z)$ is the amplitude of the power spectrum of the density perturbations.

In the background sector, we study supernovae type Ia (SNeIa) datasets and Cosmic Microwave Background (CMB) data. Union2 dataset [29] has been employed in order to study the luminosity distance to each SNeIa with a given redshift. The distance modulus, $\tilde{\mu}(z)$ (do not confuse with the structural function μ), is the difference between the apparent magnitude, m, and the absolute magnitude, M, and can be written in terms of a dimensionless cosmic time, $\tau = H_0 t$ where H_0 is the Hubble

[3] The parameter α cannot take negative values since the potential would stop being an inverse power-law

parameter today, as

$$\tilde{\mu}(\tau) = m - M = 5\log d_L(\tau) + 5\log\left(\frac{cH_0^{-1}}{Mpc}\right) + 25 \\ = 5\log d_L(\tau) + \tilde{M} \quad (27)$$

and it is directly related to the luminosity distance, $d_L(\tau) = \frac{a(\tau_0)}{a(\tau)}\int_{\tau}^{\tau_0}\frac{d\tau'}{a(\tau')}$. $\tilde{M}$ is considered as a nuissance parameter and must minimize the theoretical expression of chi-square.

Concerning the CMB data [30] we use the distance prior method which uses two distances ratios measured by means of the CMB temperature power spectrum, namely the acoustic scale, l_A, which is defined as the ratio of the angular diameter distance, $d_A(z)$, and the comoving sound horizon, $r_s(z)$, evaluated at the decoupling epoch, z_*,

$$l_A \equiv \frac{d_A(z_*)}{r_s(z_*)}, \quad (28)$$

and the shift parameter, R, given by

$$R = \sqrt{\Omega_m} H_0 d_A(z_*). \quad (29)$$

The values of the distance priors [30] are

$$\begin{pmatrix} l_A \\ R \\ z_* \end{pmatrix} = \begin{pmatrix} 302.10 \pm 0.86 \\ 1.710 \pm 0.019 \\ 1090.04 \pm 0.93 \end{pmatrix}$$

and the inverse covariance matrix

$$C^{-1} = \begin{pmatrix} 1.800 & 27.968 & -1.103 \\ 27.968 & 5667.577 & -92.263 \\ -1.103 & -92.263 & 2.923 \end{pmatrix}$$

therefore, the chi-square is given by $\chi^2 = \chi^T C^{-1} \chi$, where χ is a vector which saves the difference between the theoretical values and the observed ones of l_A, R and z_*.

Once the χ^2 analysis for the growth structure formation and the expansion history are done, one can superpose both outcomes in order to implement a total χ^2 analysis. This allows us to constrain further the parameters of the theories under study and to work with a more accurate analysis.

V. RESULTS AND MODELS COMPARISON

Results Brans-Dicke theories

The χ^2 analysis (figure **??**) reveals that either the parameter ω and α are not constrained at all, while $\Omega_{m,0} \in [0.28, 0.34]$ clearly. The best-fitting JFBD model

Survey	Redshift, z	$f\sigma_8(z)$	Reference
THF	0.02	0.40 ± 0.07	[17]
DNM	0.02	0.31 ± 0.05	[18]
6dFGS	0.07	0.42 ± 0.06	[19]
2dFGRS	0.17	0.42 ± 0.06	[20, 21]
2SLAQ	0.55	0.45 ± 0.05	[22]
SDSS LRG	0.34	0.53 ± 0.07	[23, 24]
	0.25	0.35 ± 0.06	
	0.37	0.46 ± 0.04	
BOSS	0.57	0.43 ± 0.07	[25]
WiggleZ	0.20	0.40 ± 0.13	[26]
	0.40	0.39 ± 0.08	
	0.60	0.40 ± 0.07	
	0.76	0.48 ± 0.09	
VVDS	0.77	0.49 ± 0.18	[21, 27]
VIPERS	0.80	0.47 ± 0.08	[28]

Table II: Observational data gathered from a large variety of galaxy power spectra from different surveys. Name of the survey (first column), value of the redshift at the measurement (second column), observed vale of the growth structure function (third column) and references (fourth column) are presented.

is represented in Figure **??** for the growth structure function. Since a graphic representation of the growth structure function of JFBD versus ΛCDM does not show any noticeable difference, in this case we consider to plot their relative difference in Figure **??**.

Quintessence

Inverse Power-Law model

One can see in Figure **??** the outcome of the χ^2 analysis and conclude that the observational data do not constrain that much the parameter α whereas the most likely range of matter content is $\Omega_{m,0} \in [0.20, 0.34]$.

The plot for the growth function $f(z)\sigma_{8,0}\delta(z)$ of the best-fitting IPL model versus the redshift can be seen in figure **??**. Other aspect to study is how much the above quantity deviates from ΛCDM for the same content of matter since we impose the model to behave as ΛCDM at the present time. This can be seen in Figure **??** and one realizes that the deviation increases for larger values of redshift, z.

Double Exponential Potential model

One can see in Figure **??** the outcome of the χ^2 analysis and conclude that the observational data indicate the most likely range of matter content which is $[0.31, 0.40]$ and α practically all the range.

The plot for the growth function $f(z)\sigma_{8,0}\delta(z)$ of the best-fitting 2EP model versus the redshift can be seen in Figure **??**. The deviation of 2EP from ΛCDM can be seen in Figure **??** where they seem to be parallel but with a greater contribution of 2EP model.

VI. EFT OF DE VERSUS MODEL BY MODEL ANALYSIS

In this section we will compare the efficiency, advantages and disadvantages of the data analysis performed by using the EFToDE framework in contrast with a more traditional model by model analysis.

- **EFToDE**
  - *Advantages:*
    1. Fast.
    2. Able to constrain the EFT parameter space and analyse a bunch of theories which satisfy the stability conditions.
  - *Disdvantages:*
    1. It depends on concrete parametrisation, which might be arbitrary (examples in literature).
    2. Wrong parametrisation? To compare [16] with parametrisation showed in literature.
    3. Number of parameters is high: cosmological parameters $\{\Omega_m, \Omega_r, \sigma_8, M_{Pl}\}$ and EFT parameters $\{\mu, \epsilon_4, \mu_3, ...\}$. The higher the number of parameters, the better might be the fitting curve but the higher the value of the minimum χ^2. That would mean the less realistic the model might be with respect to the observational dataset.
    4. We cannot identify the kind of theory which the best fit corresponds to (cosmological constant, quintessence, f(R) etc). Given a theory, we can identify its EFT parameters; however, given a set of EFT parameters, it is not so straightforward to recognise the theory.
- **Model by model analysis**
  - *Advantages:*
    1. It allows to study and control the subtleties and issues of each theory such as the need of a double integration for the Quintessence models.
  - *Disdvantages:*
    1. Slow.
    2. No broad picture and highly time consuming. The analysis needs to be repeated for several theories to reach a good conclusion.

In conclusion, EFToDE analysis may be fast and it allows to study a huge number of theories; but it might not take into account some important subtleties. Conversely, a model by model analysis complements the EFT study, such that it is slow but capable of dealing with computational details of each theory. It depends on what the scientist's priorities are to choose the kind of analysis we want to perform.

VII. CONCLUSIONS

They need the full plots and χ^2 values.

We have compared the predictions of competitive extended theories of gravity, namely two quintessence models, Inverse Power-Law and Double Exponential Potential models, and one Brans-Dicke theory without potential with observational measurements of luminosity distance vs. redshift from the SNIa catalogue (**which one?**), CMB data and large-scale structure growth rate surveys. We have obtained the corresponding χ^2 confidence regions in the planes $\{\alpha, \Omega_m h^2\}$ - where parameter α takes different interpretation depending on the model iunder consideration - and compared their predictions with both the Concordance ΛCDM and Dark Energy models with constant equation of state ωCDM. It is worth noting that all the models provide an expanding accelerated Universe today (see Table **??**). Nevertheless, not all of them cause the same acceleration being the JFBD model the one providing the closest value to the observational value $q-0.55$

With respect to the growth structure functions corresponding to the best fit for every model, one can observe how all the curves are convex and they all coincide at redshift $z = 0$. The one which less deviates from ΛCDM is Jordan Fierz Brans Dicke model because it has the same background evolution for the same amount of matter. In contrast, the Double Exponential Potential model shows the most distant curve. Moreover, both ωCDM and Inverse Power-Law show almost parallel curves of the growth structure function.

To summarize, it seems that the considered large-scale structure datasets (Table II) are not fully stringent for the parameters of our models.

Furthermore, we have found that the best-fitting model for the combined χ^2 turns out to be ΛCDM with a matter content of $\Omega_{m,0} = 0.302$, closely followed by the Inverse Power-Law model of Quintessence with $\Omega_{m,0} = 0.265$. One could think that the second best model would be Jordan Fierz Brans Dicke since their curves are closer to the ΛCDM ones. In fact, it is not because this model has more independent parameters to fit and therefore its χ^2 is worse than other models with less number of independent parameters.

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