When EFT of LSS meets EFT of DE

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2 Non-linear equation for the matter density contrast

Say intro stuff here :
- EFToDE : we don't touch the matter sector
- Thus : standard GR procedure to get continuity and Euler equation
- Introduce counter terms with EFToLSS procedure
- Gravitational sector is changed : EFToDE Poisson equation

2.1 Step 1 : Matter equations of motion (SPT)

→ *Pic your favourite metric.*

The perturded FLRW line element is

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + a^2(1+2\Psi)\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{1}$$

Where Φ and Ψ are resp. the perturbation of the Newtonian potential resp. curvature potential, both depend on cosmic time and space and a is the scaler factor of the universe.

The corresponding Christoffel symbols $\Gamma^\sigma_{\mu\nu}$ are

$$\begin{aligned}
\Gamma^0_{00} &= \dot{\Phi} \\
\Gamma^0_{0i} &= \partial_i \Phi \\
\Gamma^0_{ij} &= a^2\delta_{ij}(1+2\Psi-2\Phi)(H+\dot{\Psi}) \\
\Gamma^j_{00} &= \frac{1-2\Psi+2\Phi}{a^2}\partial^j\Phi \\
\Gamma^j_{0i} &= \delta^j_i(H+\dot{\Psi}) \\
\Gamma^j_{ki} &= \delta^j_k\partial_i\Psi + \delta^j_i\partial_k\Psi - \delta^{jm}\partial_m\Psi\delta_{ki},
\end{aligned}$$

where $H = \dot{a}/a$ is the Hubble rate and a dot means a derivatives w.r.t. cosmic time.

→ *Pic your favourite EMT.*

The stress-energy tensor of perfect fluid reads

$$T_{ab} = (\rho + P)u_a u_b + P g_{ab} \tag{2}$$

and the four-velocity of comoving observers is

$$u^a = \xi(1, v^i) \tag{3}$$

where ξ is the Lorentz factor and $v^i = \frac{\mathrm{dx^i}}{\mathrm{dt}}$ the components of the comoving velocity 3-vector. The four-velocity is timelike, $u_a u^a = -1$, thus one can fix the Lorentz factor to $\xi = (1-\Phi)\gamma$, being $\gamma = (1+v^2_{phys})^{-1/2}$ the special relativistic Lorentz factor and $v^2_{phys} = a^2(1+2\Psi-2\Phi)v^2$ the physical velocity.

Hence, the stress-energy tensor components read as

$$T_{00} = (\rho + P)(1 + 2\Phi)\gamma^2 - P(1 + 2\Phi), \tag{4}$$
$$T_{0i} = -a^2(\rho + P)(1 + 2\Psi)\gamma^2 v_i, \tag{5}$$
$$T_{ij} = a^4(\rho + P)(1 + 4\Psi - 2\Phi)\gamma^2 v_i v_j + a^2 P(1 + 2\Psi)\delta_{ij}. \tag{6}$$

→ *Compute the covariant conservation of EMT to get the continuity and Euler equation.*

HYP : dropping relativistic corrections ($\gamma \sim 1$) and considering cold dark matter ($P \sim 0$).

The time-like conservation equation ($\nabla_a T^{a0} = 0$) leads to

$$\dot{\rho} + \partial_m(\rho v^m) + 3\rho(H + \dot{\Psi}) = 0. \tag{7}$$

The space-like conservation equation ($\nabla_a T^{ai} = 0$) leads to

$$\dot{v^i} + v^m \partial_m v^i + 2Hv^i + \frac{\partial^i \Phi}{a^2} = 0. \tag{8}$$

→ *Quasi-static approximation and separation of perturbations and background. XXX*

One decomposes :
- $\rho(t, \mathbf{x}) \to \bar{\rho}(t) + \delta\rho(t, \mathbf{x})$
- $v^i(t, \mathbf{x}) \to v^i(t, \mathbf{x})$, by definition the background is fixed to have no velocity

and introduces :
- velocity divergence $\Theta = \partial_i v^i = \Delta\Psi$ (since $v^i = \partial^i \Psi$ and therefore, $v^i = \partial^i \Delta^{-1}\Theta$, being Δ the Laplacian operator)
- density contrast $\delta = \delta\rho/\bar{\rho}$
- the QSA : neglect time derivatives of the gravitational potentials w.r.t spatial ones.

This leads to obtaining:

$$\dot{\delta} + \Theta = -\Theta\delta - (\partial_m \Delta^{-1}\Theta)\partial^m \delta \tag{9}$$
$$\dot{\Theta} + 2H\Theta + \frac{\partial^i \Phi}{a^2} = -\partial_m \Delta^{-1}\Theta \partial_m \Theta - \partial_i \partial_m \Delta^{-1}\Theta \partial_m \partial_i \Delta^{-1}\Theta. \tag{10}$$

Note that the lhs of the equations contain O(1) terms whereas the rhs contain O(2).

2.2 Step 2 : Counter Terms (EFToLSS)

Up to this point all we have used is SPT. Nevertheless, it is known that it breaks down at some scale in the sense that predictions from the theory cannot be trusted any longer due to the uncapability of encoding short-distance physics effects. The idea is that at sufficient large scales, the components of the Universe can be considered as perfect fluids that do not interact among each other. However, the closer we get to non-linear scales, the more relevant the interactions seem to be. That is why deviations from perfect fluid behaviour appear to be crucial in our analysis. The physical understanding of the counterterms. At small scales we can think of two different effects:

1. Those kind of interactions which do not correlate with macroscopic properties, and therefore they look like random variables. We call them *stochastic term* and it represents diffusion and dissipation phenomena where exchange of energy is involved. It is possible to see that they are relevant at higher order, therefore we do not include them in our 1-loop analysis.

2. Those ones which do correlate with macroscopic properties of the fluid and give rise to *bulk* and *shear viscosities*. They look like $Z_\delta \Delta\delta$ and $Z_\Theta \Delta\Theta$, being Z_δ and Z_Θ constants.

It can be mathematically obtained buy splitting up the long-wavelength and short-wavelength modes in the stress energy tensor, and integrating out the short modes. That we will yield an effective energy-momentum tensor [**?**]. The counterterms are introduced in the equation for velocity (12) and not in (11) because matter density need to be conserved.

$$\dot{\delta} + \Theta = -\Theta\delta - (\partial_m \Delta^{-1}\Theta)\partial^m \delta \tag{11}$$

$$\dot{\Theta} + 2H\Theta + \frac{\partial^i \Phi}{a^2} = -\partial_m \Delta^{-1}\Theta\partial_m\Theta - \partial_i\partial_m\Delta^{-1}\Theta\partial_m\partial_i\Delta^{-1}\Theta - Z_\delta\Delta\delta - Z_\Theta\Delta\Theta. \tag{12}$$

2.2.1 Fourier space

The Fourier transform is given by

$$\tilde{\mathcal{O}}[\mathbf{k}] = \int \frac{d^3\mathbf{x}}{(2\pi)^{3/2}}\mathcal{O}(\mathbf{x})\mathrm{e}^{-\mathbf{i}\mathbf{k}\cdot\mathbf{x}} \quad \Leftrightarrow \quad \mathcal{O}[\mathbf{x}] = \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^{3/2}}\tilde{\mathcal{O}}(\mathbf{k})\mathrm{e}^{\mathbf{i}\mathbf{k}\cdot\mathbf{x}}. \tag{13}$$

Therefore, equation (11) in Fourier space is written as

$$\dot{\delta}_k + \Theta_k = -\int \frac{\mathrm{d}^3\mathbf{q}\mathrm{d}^3\mathbf{r}}{(2\pi)^6}(2\pi)^3\delta(\mathbf{k}-\mathbf{q}-\mathbf{r})\alpha(\mathbf{q},\mathbf{r})\Theta(\mathbf{q})\delta(\mathbf{r}) \tag{14}$$

where $\alpha(\mathbf{q},\mathbf{r}) = 1 + \dfrac{\mathbf{q}\cdot\mathbf{r}}{q^2}$

Thus, equation (12) after symmetrizing the $\partial_m\partial^{-2}\Theta\partial_m\Theta$ term reads

$$\begin{aligned}\dot{\Theta}_k + 2H\Theta_k - \frac{k^2}{a^2}\Phi_k = &+ \frac{k^2}{a^2}[Z_\delta\delta_k + Z_\Theta\Theta_k] \\ &- \int \frac{\mathrm{d}^3\mathbf{q}\mathrm{d}^3\mathbf{r}}{(2\pi)^6}(2\pi)^3\delta(\mathbf{k}-\mathbf{q}-\mathbf{r})\beta(\mathbf{q},\mathbf{r})\Theta(\mathbf{q})\Theta(\mathbf{r})\end{aligned} \tag{15}$$

where $\beta(\mathbf{q},\mathbf{r}) = \dfrac{\mathbf{q}\cdot\mathbf{r}(\mathbf{q}+\mathbf{r})^2}{2q^2r^2}$

This can be merged into one equation :

$$\begin{aligned}\ddot{\delta}_k + 2H\dot{\delta}_k + \frac{k^2}{a^2}\Phi_k = &-\int \frac{\mathrm{d}^3\mathbf{q}\mathrm{d}^3\mathbf{r}}{(2\pi)^6}(2\pi)^3\delta(\mathbf{k}-\mathbf{q}-\mathbf{r})\alpha(\mathbf{q},\mathbf{r})\left(\dot{\Theta}(\mathbf{q})\delta(\mathbf{r}) + \Theta(\mathbf{q})\dot{\delta}(\mathbf{r})\right) \\ &- 2H\int \frac{\mathrm{d}^3\mathbf{q}\mathrm{d}^3\mathbf{r}}{(2\pi)^6}(2\pi)^3\delta(\mathbf{k}-\mathbf{q}-\mathbf{r})\alpha(\mathbf{q},\mathbf{r})\Theta(\mathbf{q})\delta(\mathbf{r}) \\ &- \frac{k^2}{a^2}[Z_\delta\delta_k + Z_\Theta\Theta_k] \\ &+ \int \frac{\mathrm{d}^3\mathbf{q}\mathrm{d}^3\mathbf{r}}{(2\pi)^6}(2\pi)^3\delta(\mathbf{k}-\mathbf{q}-\mathbf{r})\beta(\mathbf{q},\mathbf{r})\Theta(\mathbf{q})\Theta(\mathbf{r})\end{aligned} \tag{16}$$

and so

$$\ddot{\delta}_k + 2H\dot{\delta}_k + \frac{k^2}{a^2}\Phi_k = -\frac{k^2}{a^2}[Z_\delta \delta_k + Z_\Theta \Theta_k]$$
$$-\int \frac{\mathrm{d}^3\mathbf{q}\mathrm{d}^3\mathbf{r}}{(2\pi)^6}(2\pi)^3\delta(\mathbf{k}-\mathbf{q}-\mathbf{r})\left\{\alpha(\mathbf{q},\mathbf{r})\left(\dot{\Theta}(\mathbf{q})\delta(\mathbf{r})+\Theta(\mathbf{q})\dot{\delta}(\mathbf{r})+2H\Theta(\mathbf{q})\delta(\mathbf{r})\right)-\beta(\mathbf{q},\mathbf{r})\Theta(\mathbf{q})\Theta(\mathbf{r})\right\} \tag{17}$$

Using (10) and (9) at linear level, the term in {} can be further reduced, dropping dependencies for clarity :

$$\alpha\left(\dot{\Theta}\delta + \Theta\dot{\delta} + 2H\Theta\delta\right) - \beta\Theta\Theta = \alpha\left(\Theta\dot{\delta} + \delta(\dot{\Theta}+2H\Theta)\right) - \beta\Theta\Theta \tag{18}$$
$$= \alpha\left(-\dot{\delta}\dot{\delta} - \frac{k^2}{a^2}\Phi\delta\right) - \beta\dot{\delta}\dot{\delta} \tag{19}$$
$$= -\alpha\frac{k^2}{a^2}\Phi\delta - \gamma\dot{\delta}\dot{\delta} \tag{20}$$
$$\tag{21}$$

where the kernel $\gamma(\mathbf{q},\mathbf{r}) = \alpha(\mathbf{q},\mathbf{r}) + \beta(\mathbf{q},\mathbf{r})$.

Thus,

$$\ddot{\delta}_k + 2H\dot{\delta}_k + \frac{k^2}{a^2}\Phi_k = -\frac{k^2}{a^2}[Z_\delta \delta_k + Z_\Theta \Theta_k] + \int \frac{\mathrm{d}^3\mathbf{q}\mathrm{d}^3\mathbf{r}}{(2\pi)^6}(2\pi)^3\delta(\mathbf{k}-\mathbf{q}-\mathbf{r})\left\{\alpha(\mathbf{q},\mathbf{r})\frac{k^2}{a^2}\Phi(q)\delta(\mathbf{r}) + \gamma(\mathbf{q},\mathbf{r})\dot{\delta}(\mathbf{q})\dot{\delta}(\mathbf{r})\right\} \tag{22}$$

2.3 Step 3 : Poisson equation (EFToDE)

The MG theories encompassed within the EFToDE formalism are those with an extra propagating degree of freedom. Therefore, one would expect new interactions due to the new scalar field. In the Jordan frame the new couplings which may appear are $\delta\Phi$, $\pi\Phi$ and $\pi\Psi$ according to the EFToDE action written in Newtonian gauge

$$S = \int a\, M^2\left[(\vec{\nabla}\Psi)^2 - 2(1+\epsilon_4)\vec{\nabla}\Phi\vec{\nabla}\Psi - 2(\mu+\mathring{\epsilon}_4)\vec{\nabla}\Psi\vec{\nabla}\pi + (\mu-\mu_3)\vec{\nabla}\Phi\vec{\nabla}\pi - \left(\mathcal{C}+\frac{\mathring{\mu}_3}{2} - \dot{H}\epsilon_4 + H\mathring{\epsilon}_4\right)(\vec{\nabla}\pi)^2\right] - a^3\,\Phi\delta\rho_m, \tag{23}$$

where $\delta\rho_m$ is the perturbation of the non-relativistic energy density, a dot means derivative w.r.t. proper time and π represents the perturbation of the scalar field. Therefore, one could expect terms of the form

$$\sim \int \frac{\mathrm{d}^3\vec{q}\mathrm{d}^3\vec{r}}{(2\pi)^6} K_1(\vec{k},\vec{q},\vec{r};z)\delta\Phi, \tag{24}$$
$$\sim \int \frac{\mathrm{d}^3\vec{q}\mathrm{d}^3\vec{r}}{(2\pi)^6} K_2(\vec{k},\vec{q},\vec{r};z)\pi\Phi, \tag{25}$$
$$\sim \int \frac{\mathrm{d}^3\vec{q}\mathrm{d}^3\vec{r}}{(2\pi)^6} K_3(\vec{k},\vec{q},\vec{r};z)\pi\Psi. \tag{26}$$

Nevertheless, the spirit of EFToDE and its unitary gauge is to hide the effect of the perturbation of the new scalar field in perturbations of the metric. We'll see that we will not obtain new contributions to the equation above but every effect from the new degree of freedom is eaten and encoded in the effective gravitational parameter $G_{\rm eff}$.

The Einstein's equations

$$M_{\rm P}^2 G_{ab} = T_{ab} \tag{27}$$

where $M_{\rm P}$ is the Planck mass, the Einstein tensor is defined as $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$ and T_{ab} is the energy-momentum tensor.

In the context of EFToDE, equations (27) read as

- **00-component**

$$M^2\left(\frac{k^2}{a^2}\left(\left(\mu-\mu_3\right)\pi - 2\Psi\left(\epsilon_4+1\right)\right) + \Phi\left(2\mathcal{C} - 6H^2\epsilon_4 - 6H^2 - 6H\mu + 6H\mu_3 + 4\mu_2^2\right) + \dot\pi\left(-2\mathcal{C} + 3H\mu - 3H\mu_3 - 4\mu_2^2\right)\right.$$
$$\left. +3\pi\left(H\left(2\mathcal{C} - H\mu + \mu^2 + \dot\mu\right) + \dot H\left(-2H\epsilon_4 - \mu + \mu_3\right)\right) - 3\dot\Psi\left(2H\epsilon_4 + 2H + \mu - \mu_3\right)\right) = \delta\rho_m \tag{28}$$

- **0i-component**

$$M^2\left(\pi\left(-2\mathcal{C} + H\mu + 2\dot H\epsilon_4 - \mu^2 - \dot\mu\right) + \Phi\left(2H\epsilon_4 + 2H + \mu - \mu_3\right) + \left(\mu_3 - \mu\right)\dot\pi + 2\dot\Psi\left(\epsilon_4+1\right)\right) = -(p_m+\rho_m)v \tag{29}$$

where v is the 3-velocity potential.

- **ij-trace component**

$$M^2\left(\frac{k^2}{a^2}\left(-\frac{2}{3}\pi\left(\epsilon_4(H+\mu) + \mu + \dot{\epsilon_4}\right) + \frac{2\Psi}{3} - \frac{2}{3}\Phi\left(\epsilon_4+1\right)\right)\right.$$
$$+\Phi\left(2\mathcal{C} + 6H^2\epsilon_4 + 6H^2 + 4H\mu - 3H\mu_3 + 2H\mu\epsilon_4 + 2H\dot{\epsilon_4} + 2\dot H\left(\epsilon_4+2\right) + 2\mu^2 + 2\dot\mu + \dot\mu_3 + \mu\mu_3\right)$$
$$+\pi\left(-2\mathcal{C}\mu - 2\dot{\mathcal{C}} + 3H^2\mu - 6\mathcal{C}H - 2H\mu^2 - 2H\dot\mu + \dot H\left(2\epsilon_4(3H+\mu) + \mu + 2\dot{\epsilon_4}\right) + 2\epsilon_4\ddot H - \mu^3 - \ddot\mu - 3\dot\mu\mu\right)$$
$$+\dot\pi\left(-2\mathcal{C} - 2H\mu + 3H\mu_3 + 2\dot H\epsilon_4 - 2\left(\mu^2+\dot\mu\right) - \dot\mu_3 - \mu\mu_3\right)$$
$$\left.+\dot\Phi\left(2H\epsilon_4 + 2H + \mu - \mu_3\right) + 2\dot\Psi\left(\left(\epsilon_4+1\right)\left(3H+\mu\right) + \dot{\epsilon_4}\right) + \left(\mu_3-\mu\right)\ddot\pi + 2\left(\epsilon_4+1\right)\ddot\Psi\right) = \delta p_m \tag{30}$$

- **ij-traceless component**

$$M^2\left(\pi\left(\epsilon_4(H+\mu) + \mu + \dot{\epsilon_4}\right) - \Psi + \Phi\left(\epsilon_4+1\right)\right) = \sigma \tag{31}$$

where σ is the scalar component of the anisotropic stress (it will be assumed to vanish for our purposes).

By combining eqs. (28) and (29) we obtain the relativistic generalisation of the Poisson equation :

- **Generalized Poisson equation**

$$M^2\left(\frac{k^2}{a^2}\left(\left(\mu-\mu_3\right)\pi - 2\Psi\left(\epsilon_4+1\right)\right) - 2\dot\pi\left(\mathcal{C} + 2\mu_2^2\right) + \Phi\left(2\mathcal{C} - 3H\mu + 3H\mu_3 + 4\mu_2^2\right) + 3\dot H\left(\mu_3-\mu\right)\pi + \left(3\mu_3 - 3\mu\right)\dot\Psi\right)$$
$$= \delta\rho_m - 3H(p_m+\rho_m)v = \rho_m\Delta_m \tag{32}$$

In the quasi-static regime (time derivatives of the metric and scalar fluctuations can be neglected with respect to spatial derivatives), it is possible to compute algebraically an effective Newton constant $G_{\rm eff}(t,k)$ of a given modified gravity theory. The entire set of perturbation equations then reduces to

$$\Delta\Phi = 4\pi G_{\rm eff}\rho_m\delta = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}H^2\Omega_m\delta\ , \tag{33}$$

where the effective gravitational constant $G_{\rm eff}$ reads

$$G_{\rm eff} \;=\; \frac{1}{8\pi M(t)^2(1+\epsilon_4)^2}\;\frac{2\mathcal{C}+\mathring{\mu}_3-2\dot{H}\epsilon_4+2H\mathring{\epsilon}_4+2(\mu+\mathring{\epsilon}_4)^2}{2\mathcal{C}+\mathring{\mu}_3-2\dot{H}\epsilon_4+2H\mathring{\epsilon}_4+2\dfrac{(\mu+\mathring{\epsilon}_4)(\mu-\mu_3)}{1+\epsilon_4}-\dfrac{(\mu-\mu_3)^2}{2(1+\epsilon_4)^2}}\ . \tag{34}$$

2.4 Step 4 : THE equation

$$\begin{aligned}\ddot{\delta}_k+2H\dot{\delta}_k+\frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}H^2\Omega_m\delta_k &= -\frac{k^2}{a^2}[Z_\delta\delta_k+Z_\Theta\Theta_k]\\ &+\int\frac{{\rm d}^3\mathbf{q}{\rm d}^3\mathbf{r}}{(2\pi)^6}(2\pi)^3\delta(\mathbf{k}-\mathbf{q}-\mathbf{r})\left\{\frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}H^2\Omega_m\alpha(\mathbf{q},\mathbf{r})\delta(\mathbf{q})\delta(\mathbf{r})+\gamma(\mathbf{q},\mathbf{r})\dot{\delta}(\mathbf{q})\dot{\delta}(\mathbf{r})\right\}\end{aligned} \tag{35}$$

And equivalently in redshift

$$\boxed{\begin{aligned}\delta''_k-\frac{1-\epsilon}{1+z}\delta'_k-4\pi G_{\rm eff}(z)\frac{\rho_M(z)}{(1+z)^2}\delta_k &= \frac{k^2}{H^2a^2}\frac{Z_\delta\delta_k+Z_\Theta H(1+z)\delta'_k}{(1+z)^2}\\ &+\int\frac{{\rm d}^3\vec{q}{\rm d}^3\vec{r}}{(2\pi)^6}(2\pi)^3\delta(\vec{k}-\vec{q}-\vec{r})\left\{4\pi G_{\rm eff}(z)\frac{\rho_M(z)}{(1+z)^2}\alpha(\vec{q},\vec{r})\delta(\vec{q})\delta(\vec{r})+\gamma(\vec{q},\vec{r})\delta'(\vec{q})\delta'(\vec{r})\right\}\end{aligned}} \tag{36}$$

where $\epsilon=-\frac{\dot{H}}{H^2}$. The right hand side corresponds to the linear equation, the first term in the left hand side is the counterterm contribution and the last term corresponds to higher corrections in perturbation theory.

3 Computing the 1 loop matter power spectrum

3.1 Step 1 : Perturbative solution of the density contrast equation

The solution of the renormalised matter density contrast up to order 3 in delta (Fig. **??**) reads as

$$\delta = \delta^{(1)}+\delta^{(2)}+\delta^{(3)}+\delta^{(CT)} \tag{37}$$

3.1.1 Order one

At late times, inside the horizon, δ_k grows according to the linear contribution in δ equation

$$\delta^{(1)}(k,z) = g_{\rm MG}(z)\delta^*_k. \tag{38}$$

To get δ^*_k from primordial initial condition we need details of both matter and radiation perturbations. In doing so we capture the evolution of the primordial gravitational potential, Φ, during matter and radiation domination. Therefore, by isotropy argument, all modes with the same k will evolve in exactly same way, thus

$$\delta^*_k \equiv T_k(z_*)\Phi_k^{Primordial} \tag{39}$$

where $T_k(z)$ is a transfer function.

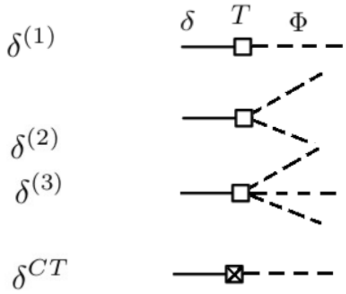

Figure 1: Diagrams of different orders in delta

3.1.2 Order two

Using Green's function method

$$\delta^{(2)}(k,z)\int_{z_*}^{z}\mathrm{d}\tilde{\mathrm{z}}\mathcal{G}(\tilde{\mathrm{z}},\mathrm{z})\,\mathit{source}^{(2)} \tag{40}$$

being $\mathit{source}^{(2)}$ the quadratic contribution in the delta equation. That would lead to

$$\delta^{(2)}(k,z)\supseteq\int\frac{\mathrm{d}^3\vec{q}\mathrm{d}^3\vec{r}}{(2\pi)^6}(2\pi)^3\delta(\vec{k}-\vec{q}-\vec{r})[\alpha(\vec{q},\vec{r})A_{\mathrm{MG}}(z)+\gamma(\vec{q},\vec{r})B_{\mathrm{MG}}(z)]\delta^*_{\vec{q}}\delta^*_{\vec{r}} \tag{41}$$

where

$$A_{\mathrm{MG}}(z)=\int_{z_*}^{z}\mathrm{d}\tilde{z}\mathcal{G}(\tilde{z},z)\frac{3}{2}\frac{G_{\mathrm{eff}}}{G_{\mathrm{N}}}(z)\ H_0^2\ \Omega_{m,0}\ (1+\tilde{z})\ g^2_{\mathrm{MG}}(\tilde{z}) \tag{42}$$

$$B_{\mathrm{MG}}(z)=\int_{z_*}^{z}\mathrm{d}\tilde{z}\mathcal{G}(\tilde{z},z)g'_{\mathrm{MG}}(\tilde{z})^2. \tag{43}$$

3.1.3 Order three

Analogously,

$$\delta^{(3)}(k,z)\int_{z_*}^{z}\mathrm{d}\tilde{\mathrm{z}}\mathcal{G}(\tilde{\mathrm{z}},\mathrm{z})\,\mathit{source}^{(2)} \tag{44}$$

where we can use the quadratic contribution of the delta equation to obtain

$$\begin{aligned}\mathit{source}^{(3)}=\int\frac{\mathrm{d}^3\vec{q}\mathrm{d}^3\vec{r}}{(2\pi)^6}(2\pi)^3\delta(\vec{k}-\vec{q}-\vec{r})[\alpha(\vec{q},\vec{r})4\pi G_{\mathrm{eff}}(z)\frac{\rho_M(z)}{(1+z)^2}(\delta^{(1)}(\vec{q})\delta^{(2)}(\vec{r})+\delta^{(2)}(\vec{q})\delta^{(1)}(\vec{r}))\\+\gamma(\vec{q},\vec{r})(\delta^{(1)'}(\vec{q})\delta^{(2)'}(\vec{r})+\delta^{(2)'}(\vec{q})\delta^{(1)'}(\vec{r}))].\end{aligned} \tag{45}$$

Plugging in the solutions for first and second order in delta, the solution at cubic order yields

$$\begin{aligned}\delta^{(3)}(k,z)\supseteq 2\int\frac{\mathrm{d}^3\vec{q}\mathrm{d}^3\vec{r}}{(2\pi)^6}(2\pi)^3\delta(\vec{k}-\vec{q}-\vec{r})\int\frac{\mathrm{d}^3\vec{p_1}\mathrm{d}^3\vec{p_2}}{(2\pi)^6}(2\pi)^3\delta(\vec{r}-\vec{p_1}-\vec{p_2})\{\alpha(\vec{q},\vec{r})[\alpha(\vec{p_1},\vec{p_2})F_{\mathrm{MG}}(z)+\gamma(\vec{p_1},\vec{p_2})G_{\mathrm{MG}}(z)]\\+\gamma(\vec{q},\vec{r})[\alpha(\vec{p_1},\vec{p_2})D_{\mathrm{MG}}(z)+\gamma(\vec{p_1},\vec{p_2})E_{\mathrm{MG}}(z)]-\beta(\vec{q},\vec{r})\alpha(\vec{p_1},\vec{p_2})J_{\mathrm{MG}}(z)\}\delta^*_{\vec{q}}\delta^*_{\vec{p_1}}\delta^*_{\vec{p_2}}\\+2\int\frac{\mathrm{d}^3\vec{q}\mathrm{d}^3\vec{r}}{(2\pi)^6}(2\pi)^3\delta(\vec{k}-\vec{q}-\vec{r})\int\frac{\mathrm{d}^3\vec{p_1}\mathrm{d}^3\vec{p_2}}{(2\pi)^6}(2\pi)^3\delta(\vec{q}-\vec{p_1}-\vec{p_2})[\alpha(\vec{q},\vec{r})\beta(\vec{p_1},\vec{p_2})-\gamma(\vec{q},\vec{r})\alpha(\vec{p_1},\vec{p_2})]J_{\mathrm{MG}}(z)\delta^*_{\vec{r}}\delta^*_{\vec{p_1}}\delta^*_{\vec{p_2}}\end{aligned} \tag{46}$$

The functions are given by

$$D_{\rm MG}(z) = \int_{z*}^{z} {\rm d}\tilde{z}\mathcal{G}(\tilde{z},z) g'_{\rm MG}(\tilde{z}) A'_{\rm MG}(\tilde{z}) \tag{47}$$

$$E_{\rm MG}(z) = \int_{z*}^{z} {\rm d}\tilde{z}\mathcal{G}(\tilde{z},z) g'_{\rm MG}(\tilde{z}) B'_{\rm MG}(\tilde{z}) \tag{48}$$

$$F_{\rm MG}(z) = \int_{z*}^{z} {\rm d}\tilde{z}\mathcal{G}(\tilde{z},z) 4\pi G_{\rm eff}(z) \frac{\rho_M(z)}{(1+\tilde{z})^2} g_{\rm MG}(\tilde{z}) A_{\rm MG}(\tilde{z}) \tag{49}$$

$$G_{\rm MG}(z) = \int_{z*}^{z} {\rm d}\tilde{z}\mathcal{G}(\tilde{z},z) 4\pi G_{\rm eff}(z) \frac{\rho_M(z)}{(1+\tilde{z})^2} g_{\rm MG}(\tilde{z}) B_{\rm MG}(\tilde{z}) \tag{50}$$

$$J_{\rm MG}(z) = \frac{1}{2}\int_{z*}^{z} {\rm d}\tilde{z}\mathcal{G}(\tilde{z},z) g'_{\rm MG}(\tilde{z})^2 g_{\rm MG}(\tilde{z}). \tag{51}$$

3.1.4 Counterterm

$$\delta^{CT}(k,z) = \frac{k^2}{H^2 a^2} c^2_{CT}(z)\delta^*_k \tag{52}$$

where

$$c^2_{CT}(z) = \int {\rm d}\tilde{z}\mathcal{G}(\tilde{z},z)\frac{Z_\delta(\tilde{z}) g_{\rm MG}(\tilde{z}) + Z_\Theta(\tilde{z}) H(1+\tilde{z}) g'_{\rm MG}(\tilde{z})}{(1+\tilde{z})^2}, \tag{53}$$

being $\mathcal{G}(\tilde{z},z)$ the Green's function.

3.2 Step 2 : Growth and transfer functions

The Green function satisfies the following equation (from the linear equation of δ):

$$\frac{{\rm d^2G(z,\tilde{z})}}{{\rm dz^2}} - \frac{1-\epsilon}{1+z}\frac{{\rm dG(z,\tilde{z})}}{{\rm dz}} - 4\pi G_{\rm eff}(z)\frac{\rho_M(z)}{(1+z)^2} G(z,\tilde{z}) = \delta(z-\tilde{z}) \tag{54}$$

$$\begin{aligned} G(z=\tilde{z},\tilde{z}) &= 0 \\ G'(z=\tilde{z},\tilde{z}) &= 1 \end{aligned} \tag{55}$$

where

$$\epsilon = -\frac{\dot{H}}{H^2} \equiv \frac{3}{2}\Omega_M(z) \tag{56}$$

and

$$\Omega_M(z) = \frac{\Omega_{m0}(1+z)^3}{\Omega_{m0}(1+z)^3 + 1 - \Omega_{m0}} \tag{57}$$

being $\Omega_{m0} + \Omega_{DE0} \equiv 1$, since $H^2 = H_0^2(\Omega_{m0}(1+z)^3 + \Omega_{DE0})$.

$$\frac{{\rm d^2A_{MG}(z)}}{{\rm dz^2}} - \frac{1-\epsilon}{1+z}\frac{{\rm dA_{MG}(z)}}{{\rm dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z) A_{\rm MG}(z) = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ g^2_{\rm MG}(z) \tag{58}$$

$$\frac{{\rm d^2B_{MG}(z)}}{{\rm dz^2}} - \frac{1-\epsilon}{1+z}\frac{{\rm dB_{MG}(z)}}{{\rm dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z) B_{\rm MG}(z) = g'^{\,2}_{\rm MG}(z) \tag{59}$$

$$\frac{{\rm d^2D_{MG}(z)}}{{\rm dz^2}} - \frac{1-\epsilon}{1+z}\frac{{\rm dD_{MG}(z)}}{{\rm dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z) D_{\rm MG}(z) = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ g'_{\rm MG}(z) A'_{\rm MG}(z) \tag{60}$$

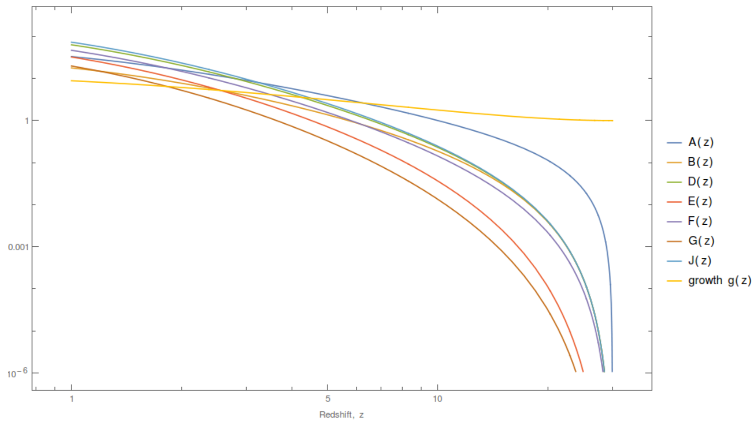

Figure 2: Transfer function and coefficients for GR.

$$\frac{\mathrm{d}^2\mathrm{E_{MG}(z)}}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{dE_{MG}(z)}}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)E_{\rm MG}(z) = g'_{\rm MG}(z)B'_{\rm MG}(z) \tag{61}$$

$$\frac{\mathrm{d}^2\mathrm{F_{MG}(z)}}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{dF_{MG}(z)}}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)F_{\rm MG}(z) = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ g_{\rm MG}(z)A_{\rm MG}(z) \tag{62}$$

$$\frac{\mathrm{d}^2\mathrm{G_{MG}(z)}}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{dG_{MG}(z)}}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)G_{\rm MG}(z) = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ g_{\rm MG}(z)B_{\rm MG}(z) \tag{63}$$

$$\frac{\mathrm{d}^2\mathrm{J_{MG}(z)}}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{dJ_{MG}(z)}}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)J_{\rm MG}(z) = g'_{\rm MG}(z)^2 g_{\rm MG}(z) \tag{64}$$

The initial conditions for all these equations are equal to zero at $z = z*$, $A = B = D = E = F = G = J = 0$, so are their derivatives with respect to redshift.
The growth function $g_{\rm MG}$

$$\frac{\mathrm{d}^2 g_{\rm MG}(z)}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{d}g_{\rm MG}(z)}{\mathrm{dz}} - -\frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)g_{\rm MG}(z) = 0 \tag{65}$$

$$\begin{aligned} g_{\rm MG}(z=z*) &= 1 \\ g'_{\rm MG}(z=z*) &= 0 \end{aligned} \tag{66}$$

3.3 Step 3 : Correlation functions and matter power spectrum

The renormalised 1 loop matter power spectrum (Fig. **??**) is

$$P_{1-\rm loop} = P_{11} + P_{13} + P_{22} + P_{\rm CT} \tag{67}$$

In order to obtain the linear power spectrum, quadratic, cubic and counter-term contributions we need to compute the following correlation functions

$$< \delta^{(1)}(k,z)\delta^{(1)}(k',z) >= (2\pi)^3\delta_D(\vec{k}+\vec{k}')P_{11}(k,z), \tag{68}$$

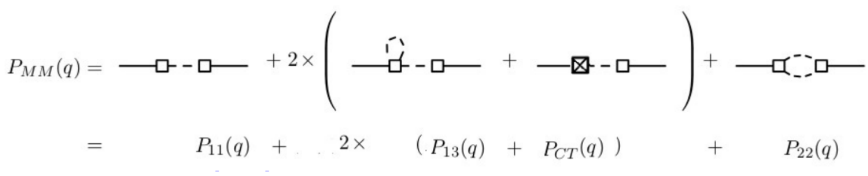

Figure 3: Renormalised 1 loop matter power spectrum.

$$< \delta^{(2)}(k,z)\delta^{(2)}(k',z) >= (2\pi)^3\delta_D(\vec{k}+\vec{k}')P_{22}(k,z), \tag{69}$$

$$< \delta^{(1)}(k,z)\delta^{(3)}(k',z) >= (2\pi)^3\delta_D(\vec{k}+\vec{k}')P_{13}(k,z), \tag{70}$$

and

$$< \delta^{(1)}(k,z)\delta^{CT}(k',z) >= (2\pi)^3\delta_D(\vec{k}+\vec{k}')P_{CT}(k,z). \tag{71}$$

Computations yield the following expressions:

$$\boxed{P_{11}(k,z) = g_{\rm MG}(z)^2\mathcal{P}_R(k)} \tag{72}$$

$$\boxed{P_{CT}(k,z) = c_{CT}(z)^2\frac{k^2}{a^2H^2}g_{\rm MG}(z)\mathcal{P}_R(k)} \tag{73}$$

$$\boxed{\begin{aligned} P_{22}(k,z) = 2\int \frac{{\rm d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})\mathcal{P}_R(\vec{k}-\vec{q})[\alpha(\vec{q},\vec{k}-\vec{q})A_{\rm MG}(z)+\gamma(\vec{q},\vec{k}-\vec{q})B_{\rm MG}(z)] \\ \times[\alpha(-\vec{q},\vec{q}-\vec{k})A_{\rm MG}(z)+\gamma(-\vec{q},\vec{q}-\vec{k})B_{\rm MG}(z)] \end{aligned}} \tag{74}$$

$$\boxed{\begin{aligned} P_{13}(k,z) = 8g_{\rm MG}(z)\mathcal{P}_R(k)\int \frac{{\rm d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})[\overline{\alpha}(\vec{k},-\vec{q})\overline{\alpha}(\vec{k}-\vec{q},\vec{q})[F_{\rm MG}(z)+2J_{\rm MG}(z)] \\ +\overline{\gamma}(\vec{k},-\vec{q})\overline{\alpha}(\vec{k}-\vec{q},\vec{q})G_{\rm MG}(z) \\ +\overline{\alpha}(\vec{k},-\vec{q})\overline{\gamma}(\vec{k}-\vec{q},\vec{q})[D_{\rm MG}(z)-2J_{\rm MG}(z)] \\ +\overline{\gamma}(\vec{k},-\vec{q})\overline{\gamma}(\vec{k}-\vec{q},\vec{q})E_{\rm MG}(z) \\ +\alpha(\vec{k}-\vec{q},\vec{q})[\beta(\vec{k},-\vec{q})-\overline{\alpha}(\vec{k},-\vec{q})]J_{\rm MG}(z)] \end{aligned}} \tag{75}$$

where overline kernels refer to symmetrised versions.

3.4 Step 4 : UV divergences

Loop integrals lead to some divergences in the ultra violet regime. In the case of EFToLSS (both in real and redshift space), we distinguish between two regimes, namely: linear regime within SPT yields reliable results and it is associated to the scale k_*, and the mild non-linear regime $k_* < k < \Lambda$ in which SPT cannot be longer trusted. In this latter regime, loop integrals do not blow up but their computations are no longer accurate if compare to observations. Therefore, CTs play their role subtracting such a divergence.

- *Local effects:* they come from high energy portions of loop integrations (mild non-linear regime). Locality is manifest by analytic terms in momentum space. The UV divergences are not predictable from the effective theory and they do not imply any physical consequences since they are absorbed by re-normalisation parameters. Those parameters are not predicted by the theory. Moreover, they encode our ignorance of high energy physics. They should emerge from either an ultimate high energy theory, or measured experimentally, or fixed by simulations.

- *Non-local effects:* they come from low energy portions of loop integrations (linear regime, where SPT plays a role). Non locality is manifest by non-analytic behaviour in momentum space. The structure of such terms differs from local ones. Moreover, their parameters are cut-off independent, therefore they could be predicted by the effective theory.

At 1-loop corrections and dropping $(k/k_{NL})^4$ terms, only P_{13} terms are renormalised (recall P_{22} is relevant at 2-loop by the stochastic term due to its k dependence). P_{13} (75) can be rewritten as

$$\begin{aligned} P_{13}(k,z) = 8g_{\rm MG}(z)\mathcal{P}_R(k)\{I_{\overline{\alpha\alpha}}(\Lambda)[F_{\rm MG}(z)+2J_{\rm MG}(z)] + I_{\overline{\alpha\gamma}}(\Lambda)[D_{\rm MG}(z)-2J_{\rm MG}(z)] \\ + I_{\overline{\gamma\alpha}}(\Lambda)G_{\rm MG}(z) + I_{\overline{\gamma\gamma}}(\Lambda)E_{\rm MG}(z) + [I_{\alpha\beta}(\Lambda) - I_{\alpha\overline{\alpha}}(\Lambda)]J_{\rm MG}(z)\}. \end{aligned} \tag{76}$$

The idea is to split the integrals up in the linear regime where SPT can be trusted, and the mild non-linear regime where a Taylor expansion can be performed since $k/k_{NL} << 1$:

$$\begin{aligned} I_{\overline{\alpha\alpha}}(\Lambda) &= \int^{\Lambda} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})\overline{\alpha}(\vec{k},-\vec{q})\overline{\alpha}(\vec{k}-\vec{q},\vec{q}) \\ &= \underbrace{\int_0^{k_*} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})\overline{\alpha}(\vec{k},-\vec{q})\overline{\alpha}(\vec{k}-\vec{q},\vec{q})}_{\text{Linear regime, SPT, }\Lambda\text{-independent}} + \underbrace{\int_{k_*}^{\Lambda} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})\overline{\alpha}(\vec{k},-\vec{q})\overline{\alpha}(\vec{k}-\vec{q},\vec{q})}_{\text{Mild non-linear regime, UV sensitive}} \\ &= a_1(\Lambda)\cdot \underbrace{k^2}_{\text{Analytic}} + b_1 \cdot \underbrace{k^3}_{Non-analytic} + O(k^4). \end{aligned} \tag{77}$$

Analytic k-dependence is linked to terms polynomial in k^2. Non-analytic terms are manifest by logarithms or fractional powers of k^2 and it is straightforward to see that they decouple from the cut-off. Therefore, by inspection of the integrals:

$$P_{13}(k,z) = 8g_{\rm MG}(z)\mathcal{P}_R(k)\{(a(\Lambda)h_1(z)\cdot k^2 + h_2(z)\cdot k^3 + O(k^4)\} \tag{78}$$

where

$$h_1(z) = \frac{4\pi}{15}\{-18D_{\rm MG}(z) - 28E_{\rm MG}(z) + 7F_{\rm MG}(z) + 2G_{\rm MG}(z) + 38J_{\rm MG}(z)\}, \tag{79}$$

$$h_2(z) = b_1[F_{\rm MG}(z)+2J_{\rm MG}(z)] + b_2[D_{\rm MG}(z)-2J_{\rm MG}(z)] + b_3G_{\rm MG}(z) + b_4E_{\rm MG}(z) + b_5J_{\rm MG}(z) \tag{80}$$

b_i coefficients can be obtained by fitting (78) to a cubic polynomial, and

$$a(\Lambda) = \int^{\Lambda} \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\frac{\mathcal{P}_R(\vec{q})}{q^2} \tag{81}$$

which presents ultra-violet sensitivity and needs to be renormalised.

3.5 Step 5 : IR divergences

In addition, P_{13} and P_{22} are separately IR divergent. Fortunately, the sum of them is free of such divergences [?] in real space.

3.6 Step 6 : Renormalisation

The re-normalisation process is similar to that of the typical scattering process at 1-loop in Field Theory [?].The renormalised 1 loop matter power spectrum (Fig. ??) is

$$P_{1-\text{loop}} = P_{11} + P_{13} + P_{22} + P_{\text{CT}} \tag{82}$$

In fact, $P_{CT}(k,z)$ and P_{13} share the same operator structure, such that all the UV divergences can be renormalised. Recall P_{22} renormalisation will be relevant at 2-loop by the stochastic term since we have decided to truncate our expansion up to order four in k/k_{NL}. Therefore, using (78)

$$P_{1-loop}(k,z) = g_{\text{MG}}(z)^2 \mathcal{P}_R(k)\left[1 + \left(\frac{8h_1(z)}{g_{\text{MG}}(z)} \times a(\Lambda) + \frac{c_{CT}^{MG}(z)^2}{a^2 H^2 g_{\text{MG}}(z)}\right) k^2 + \frac{8h_2(z)}{g_{\text{MG}}(z)} k^3\right]. \tag{83}$$

In the UV limit, terms in round brackets have no cut-off dependence since the counter-terms absorb the divergence. That means, in that limit, the factor coming from the counter-term has the same z-evolution as $f_1^{MG}(z) \equiv \frac{8h_1(z)}{g_{\text{MG}}(z)}$ up to a constant. Therefore, the previous expression can be rewritten as

$$\boxed{P_{1-loop}(k,z) = g(z)^2 \mathcal{P}_R(k)\left[1 + c_s^2\, f_1^{MG}(z)\, k^2 + f_2^{MG}(z)\, k^3 + O(k^4)\right]} \tag{84}$$

where $f_2^{MG}(z) \equiv \frac{8h_2(z)}{g_{\text{MG}}(z)}$ and c_s^2 is the **renormalisation factor** with o UV sensitivity and which needs to be fit by observational data or simulations since cannot be predicted by the effective theory. The renormalisation factor encodes all our ignorance from short distance physics, therefore it can only be predicted by an ultimate high-energy theory.

4 Comparison to CAMB HALOFit matter power spectrum

4.1 Impact of Geff on Growth functions

Comparison with GR and different cosmologies.

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