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Project proposal

When the EFT of LSS meets the EFT of DE

Contents

1 Introduction

Powerful Solar System and astrophysical tests impose stringent limits on modified gravity. Realistic models must incorporate "screening" mechanisms that ensure convergence to General Relativity on small scales and/or high-density environments. Modified gravity models such as Horndeski and Galileon theories mainly rely on the Vainshtein mechanism which is now quite well understood in a cosmological time dependent set-up. However a pertinent modelling and parametrisation of screening mechanisms are yet to be achieved. To attend to this issue, in this combined project, we would like to merge two effective field descriptions used in cosmology: (i) The effective field theory of large scale structures (EFToLSS) that allows a better description of the mildly non linear regime of perturbation theory. (ii) The effective field theory of dark energy (EFToDE) that provides a unifying description of one extra scalar modified gravity models. Doing so one could be able to extract information from the non linear corrections on the power spectrum provided by the EFToLSS in a modified gravity set-up to extract a parametrisation of its screening mechanism. The use of the EFToDE in opposition to a covariant Galileon theory for example would provide three benefits that are, testing several MG models in the same framework, set the background to be that of ΛCDM as in EFToLSS, encode all the modifications of gravity in the perturbed sector in two functions, the effective gravitational constant $G_{\rm eff}$ and the gravitational slip parameter η. Likewise, using EFToLSS instead of Standard Perturbation Theory (SPT) offers several advantages. EFToLSS is able to solve some issues present within SPT formalism. For instance, the expansion parameter is well defined, deviations from perfect fluid behaviour are considered and predictions are cut-off independent and so they are physical.
This proposal report is organised as follows. First, we give brief introductions to the effective field theory of large scale structures and to the effective field theory of dark energy. Then we present where modified gravity effects are expected arise in the computation of the matter power spectrum. We conclude by summarising the steps to follow in this project and its prospects.

2 A short guide to the Effective Field Theory of Large Scale Structures

We are interested in the study of the cosmological Large Scale Structure (LSS) formation. We would like to take into account back-reaction from small scales. Using an effective theory enables a description of a system that captures all the relevant degrees of freedom and describes relevant physics at macroscopic scale of interest. In addition, the size of non-linear terms grows with the size of fluctuations, which implies that the Universe should be described at long distances by some weakly coupled degree of freedom that becomes more interacting as we move closer to the non-linear scale, k_{NL}, where fluctuations become strongly coupled. This scaling suggest the existence of an EFT which allows to describe with arbitrary precision the Universe on IR scales, $k << k_{NL}$. At large scales, the Universe can be described by a fluid with small perturbations and the equations of motion can be organised in a derivative expansion in k/k_{NL}.

The Standard (Euclidean) Perturbation Theory (SPT from now on), applied to matter cosmological perturbations, organises the density power spectrum in a loop expansion in the initial power spectrum of inhomogeneities [2]. Nevertheless, this standard approach is not satisfactory since it presents several issues: the initial power spectrum of inhomogeneities is a *poor-defined expansion parameter*, short-scale non-linearities induce *deviations from a perfect pressure-less fluid*, which are not taken into account within SPT, and the appearance of divergences leads to *unphysical predictions*.

These three issues are well cured within the Effective Field Theory of Large Scale Structures, EFToLSS [8, 1]. Within this formalism, the short-scale modes are integrated out (or coarse grained). Then, the expansion parameters are well defined: *smoothed density* and *velocity*. Moreover, the EFT approach claims that all the terms compatible with the symmetries of the physical system should be present. Therefore, there appear additional terms, such as non-vanishing speed of sound, dissipative corrections and stochastic noise which encode the effects from the small scales. In addition, these effective corrections act as counter-terms and renormalise the theory in such a way that predictions are physical. In other words, the counter-terms cancel the UV divergences and the quantities remain cut-off independent.

2.1 The equations of motion

EFToLSS is purely General Relativity (GR), therefore the metric is given by

$$\mathrm{ds}^2 = -\mathrm{e}^{2\Psi(\mathrm{t},\mathbf{x})}\mathrm{dt}^2 + \mathrm{a(t)}^2\mathrm{e}^{2\Phi(\mathrm{t},\mathbf{x})}\mathrm{d}\mathbf{x}^2, \tag{1}$$

where $\mathrm{d}\mathbf{x}^2 = \mathrm{d}\chi^2 + \chi^2[\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2]$ in co-moving spherical coordinates for a flat Universe, and Φ and Ψ are the so-called Bardeen potentials. The Einstein equations are given by

$$M_{\rm P}^2 G_{ab} = T_{ab} \tag{2}$$

where $M_{\rm P}$ is the Planck mass, the Einstein tensor is defined as $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$. We assume perfect fluid behaviour, that is, $T_{ab} = (\rho + P)u_a u_b + Pg_{ab}$, where the four-velocity of co-moving observers is $u^a = \xi(1, v^i)$. ξ is the Lorentz factor and $v^i = \frac{\mathrm{dx^i}}{\mathrm{dt}}$ the velocity vector. The Lorentz factor is given by $\xi = \mathrm{e}^{-\Psi}\gamma$, being $\gamma = (1+v_{phys}^2)^{-1/2}$ the special relativistic Lorentz factor and $v_{phys}^2 = a^2 \mathrm{e}^{2(\Phi-\Psi)} v^2$ the physical velocity.

Considering covariant conservation of T_{ab}, linear order in the gravitational potentials and dropping relativistic corrections ($\gamma \simeq 1$), Cold Dark Matter (CDM) component, i.e. $P = 0$, and using the absence of anisotropic stress argument, the fluid equations read as

$$\partial_t \rho + \partial_m(\rho v^m) + 3\rho(H + \dot{\Phi}) = 0, \tag{3}$$

$$\partial_t v^i + v^m \partial_m v^i + 2Hv^i + \frac{\partial^i \Psi}{a^2} = 0, \tag{4}$$

$$\frac{\partial^2 \Phi}{a^2} + \frac{\delta\rho}{2M_p^2} = 0. \tag{5}$$

2.2 Cosmological matter perturbations

The goal is to obtain the equation for cosmological matter density perturbations. We write equations above in terms of the density contrast, $\delta = \delta\rho/\rho_0$ where ρ_0 is the mean value of the matter density. Besides, the velocity divergence, Θ, is defined as the Laplacian of the gravitational potential Φ, i.e. $\Theta = \partial_i v^i = \partial^2\Phi$. Therefore, the 3-velocity can be written as $v^i = \partial^i \Phi = \partial^i \partial^{-2}\Theta$. In addition, *Quasi-static Approximation* (QSA) is assumed, that is, time derivative terms for the gravitational potentials are discarded, and only those including density perturbations are kept. Finally, the fluid equations reduces to the density contrast equation

$$\dot{\delta} + \Theta = -\Theta\delta - (\partial_m \partial^{-2}\Theta)\partial^m \delta, \tag{6}$$

and the velocity divergence equation

$$\dot{\Theta} + \partial_m \partial^{-2}\Theta \partial_m \Theta + \partial_i \partial_m \partial^{-2}\Theta \partial_m \partial_i \partial^{-2}\Theta + 2H\Theta + \frac{3}{2}H^2\Omega_M(z)\delta = 0. \tag{7}$$

One would need to add the most general combination of next-order operators as counter-terms to equation (7), namely $Z_\delta \frac{\partial^2\delta}{a^2}$ and $Z_\Theta \frac{\partial^2\Theta}{a^2}$ [3]. These counter-terms are to absorb UV divergences at 1-loop and they mean to be one of the main differences with SPT. Likewise, one could add a *stochastic term* (sub-dominant at 1-loop correction) which removes the UV divergences at 2-loop order.

It would be more convenient to work in Fourier space and to write equations as functions of redshift. Working out some calculations, one could rearrange (6) and (7) in a single equation for the density contrast. Therefore, the non-linear equation for the matter density contrast in Fourier space to 1-loop correction, $O(\delta^3)$,

$$\boxed{\begin{aligned} \delta_k'' - \frac{1-\epsilon}{1+z}\delta_k' - \frac{3}{2}\frac{\Omega_M(z)}{(1+z)^2}\delta_k = {} & \frac{k^2}{H^2a^2}\frac{Z_\delta\delta_k + Z_\Theta H(1+z)\delta_k'}{(1+z)^2} \\ & + \int \frac{\mathrm{d}^3\vec{q}\mathrm{d}^3\vec{r}}{(2\pi)^6}(2\pi)^3\delta(\vec{k}-\vec{q}-\vec{r})[\alpha(\vec{q},\vec{r})\frac{3}{2}\frac{\Omega_M(z)}{(1+z)^2}\delta(\vec{q})\delta(\vec{r}) \\ & + \gamma(\vec{q},\vec{r})\delta'(\vec{q})\delta'(\vec{r})] \end{aligned}} \tag{8}$$

where $\epsilon = -\frac{\dot{H}}{H^2}$ and $' \equiv \mathrm{d/dz}$. The LHS of this equation will lead the linearised equation, which is k independent. The first line in the RHS is counter-term (CT) contribution and the rest is the quadratic part. Kernels α and γ multiply symmetric combinations of δ and δ', respectively. This implies that from subsequent expressions, it will be only possible to get symmetric combinations of $\vec{q}$ and $\vec{r}$. Therefore, both kernels, α and $\gamma \equiv \alpha + \beta$, can be replaced by their symmetrised versions. Thus

$$\alpha(\vec{q},\vec{r}) \to \frac{1}{2}(\vec{q}+\vec{r})\cdot\left(\frac{\vec{q}}{q^2}+\frac{\vec{r}}{r^2}\right) \tag{9}$$

and

$$\beta(\vec{q},\vec{r}) \to \frac{\vec{q}\cdot\vec{r}}{2q^2r^2}(\vec{q}+\vec{r})^2. \tag{10}$$

Finally, one would need to obtain the solutions for the linear equation, the quadratic, cubic and counter-term contributions, that are $\delta^{(1)}$, $\delta^{(2)}$, $\delta^{(3)}$ and $\delta^{(CT)}$ (please, refer to appendix A).

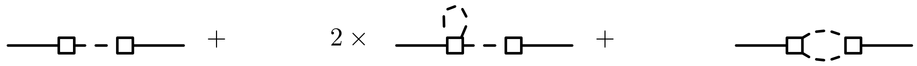
2.3 Matter Power spectrum

Given the solution of the density contrast, the power spectrum is obtained from

$$< \delta^*(k,z)\delta(k',z) >= (2\pi)^3\delta_D(\vec{k}+\vec{k}')P(k,z), \tag{11}$$

where the *ergodic theorem* is used. The matter power spectrum at 1-loop [3] should present the following expression:

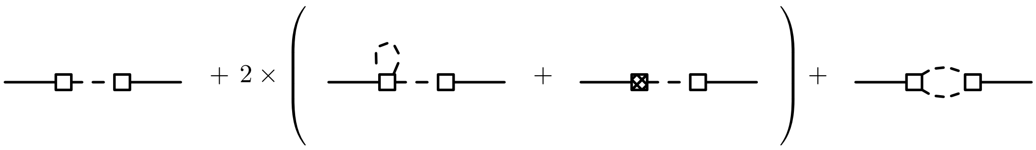
$$P_{MM}(q) = \quad P_{11}(q) \quad + \quad P_{13}(q) \quad + \quad P_{22}(q) \tag{12}$$



(13)

Refer to appendix A for detailed form of the power spectrum contributions. The diagram of the renormalised matter power spectrum is given by

$P_{MM}(q) =$



(14)

where the counter-term contribution is added to cancel the cut-off dependence, of P_{13} [1]. Indeed, $P_{CT}(k,z)$ has the same operator structure of P_{13} such that all the UV divergences can be renormalised. Recall P_{22} is renormalised at 2-loop by the stochastic term. The re-normalisation process is similar to that of the typical scattering process at 1-loop in Field Theory [9]. Focusing on low-k behaviour, one can perform a Taylor expansion around $k = 0$. Moreover, the 1-loop power spectrum prediction would read as

$$P_{1-loop}(k,z) = g(z)^2\mathcal{P}_R(k)\left[1+\mu\Xi(z)k^2\right] \tag{15}$$

where $\mathcal{P}_R(k) = \frac{A_s}{k^3}\left(\frac{k}{k_0}\right)^{n_s-1}$ is the primordial power spectrum . The next question is how to fix the factor μ, which is just a number with no cut-off dependence (see appendix A fot further details). One can use **CAMB** code to fit the coefficients.

2.4 Divergences

One would need to be aware of divergences which come out of the loop diagrams and of possible predictions in the effective field theory [5]. Distinguishing among high energy and low energy effects could seem to be difficult. Nevertheless, those terms sharing the structure of counter-terms refer to local effects. Otherwise, they are related to non-local effects.

- *Local effects:* they come from high energy portions of loop integrations. Locality is manifest by analytic terms in momentum space. The UV divergences are not predictable from the effective theory and they do not imply any physical consequences since they are absorbed by re-normalisation parameters. Those parameters are not predicted by the theory. Moreover, they encode our ignorance of high energy physics. They should emerge from either an ultimate high energy theory, or measured experimentally, or fixed by simulations.

- *Non-local effects:* they come from low energy portions of loop integrations. Non locality is manifest by non-analytic behaviour in momentum space. The structure of such terms differs from local ones. Moreover, their parameters are cut-off independent, therefore they could be predicted by the effective theory.

In the case of our concern, it is possible to spot those kind of behaviours. Equation (92) could be rewritten as

$$P_{13}(k,z) = 4g(z)\mathcal{P}_R(k)\left[I_{\alpha\alpha}(\Lambda)F(z) + I_{\alpha\gamma}(\Lambda)D(z) + I_{\gamma\alpha}(\Lambda)E(z) + I_{\gamma\gamma}(\Lambda)G(z)\right]. \tag{16}$$

For example,

$$\begin{aligned} I_{\alpha\alpha}(\Lambda) &= \int^{\Lambda}\frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})\alpha(\vec{k},-\vec{q})\alpha(\vec{k}-\vec{q},\vec{q}) \\ &= \underbrace{\int_0^{k_*}\frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})\alpha(\vec{k},-\vec{q})\alpha(\vec{k}-\vec{q},\vec{q})}_{k_* << k \text{ regime, } \Lambda\text{-independent}} + \underbrace{\int_{k_*}^{\Lambda}\frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})\alpha(\vec{k},-\vec{q})\alpha(\vec{k}-\vec{q},\vec{q})}_{k/k_* << 1 \text{ Taylor expansion, } \Lambda\text{-dependent}} \\ &= \underbrace{\underbrace{a_1(\Lambda)}_{\text{fixed by renormalisation}}\cdot k^2}_{\text{analytic behaviour, UV sensitive}} + \underbrace{\underbrace{b_1}_{\text{low-energy}}\cdot k^3}_{Non-analytic} + O(k^4). \end{aligned} \tag{17}$$

Analytic k-dependence is linked to terms polynomial in k^2. Non-analytic terms are manifest by logarithms or fractional powers of k^2 and it is straightforward to see that they decouple from the cut-off. Therefore, by inspection of the integrals:

$$\begin{aligned} P_{13}(k,z) = 4g(z)\mathcal{P}_R(k)[&(a_1(\Lambda)F(z) + a_2(\Lambda)D(z) + a_3(\Lambda)E(z) + a_4(\Lambda)G(z)) \cdot k^2 \\ &+ (b_1F(z) + b_2D(z) + b_3E(z) + b_4G(z)) \cdot k^3 \\ &+ O(k^4)]. \end{aligned} \tag{18}$$

In addition, P_{13} and P_{22} are separately IR divergent. Fortunately, the sum of them is free of such divergences [3].

3 A short guide to the Effective Field Theory of Dark Energy

The effective field theory of dark energy (EFToDE) was developed [7, 6] by using the formalism of the EFT of inflation [4]. The idea is to treat cosmological perturbations as the Goldstone boson of spontaneously broken time-translations [11]. Doing so, the EFT of DE formalism gives a frame theory that encompasses all the modified gravity models that contain only one extra scalar field in Einstein's field equation.

More specifically, the EFToDE parametrizes theories in terms of structural functions of time. In this way the confrontation of theories with observations can be made directly in the phase space of theories. However, in this case the observations must be able to fix continuous functions of time instead of only estimating amplitudes of parameters characterizing the different theories. Pertinent phenomenological parametrisations must thus be implemented. EFToDE is often treated in the unitary gauge, that has the advantage of inducing operators in powers of the number of perturbations. Thereby operators beyond the first order will only play a role in the perturbation sector. This separation between background and perturbation operators allows one to fully set the background of an EFT model to any given background.

3.1 The action

The action (19) is written in terms of the metric to which baryonic matter fields and dark matter fields, contained inside the action S_m, are minimally coupled, that is to say there are no derivatives acting on the matter fields. This display is called the Jordan frame in opposition to the Einstein frame where the matter fields are non-minimally coupled and the scalar field is present in the metric.

$$\begin{aligned} S = S_m[g_{\mu\nu}, \Psi_i] + \int d^4x\sqrt{-g}\,\frac{M^2(t)}{2}\,\big[R - 2\lambda(t) - 2\mathcal{C}(t)g^{00} \\ - \mu_2^2(t)(\delta g^{00})^2 - \mu_3(t)\,\delta K\delta g^{00} + \epsilon_4(t)\left(\delta K^\mu_\nu\,\delta K^\nu_\mu - \delta K^2 + \frac{{}^{(3)}\!R\,\delta g^{00}}{2}\right)\bigg]\ , \end{aligned} \tag{19}$$

Moreover, the action is written in the unitary gauge, where the time coordinate t is fixed to be proportional to the scalar field while the three space coordinates are left undetermined. This choice of gauge induces the presence of non-covariant terms, namely the perturbations of the lapse component of the metric $\delta g^{00} \equiv g^{00} + 1$, the perturbation of the extrinsic curvature on the $t = const.$ hyper-surfaces $\delta K_{\mu\nu}$ and its trace δK, the three dimensional Ricci scalar ${}^{(3)}R$ computed on such a hyper-surface. This choice of gauge also explains why the scalar field does not appear, its dynamics is entirely encoded in the metric's degrees of freedom.

The first line of the action (19) contains only the operators (couplings) that contribute to the evolution of the background: $M^2(t)$, $\lambda(t)$ and $\mathcal{C}(t)$. The second line contains all the terms contributing on the perturbation sector: $\mu_2^2(t)$, $\mu_3(t)$ and $\epsilon_4(t)$ are mass terms. The scale of the coefficients is set in powers of the Hubble parameter, *i.e* $\mathcal{C} \sim H^2$, $\mu_3 \sim H$, $\epsilon_4 \sim 1$. The background couplings play a role in the expansion history and can play a role in the perturbation sector. However, the perturbation operators play solely a role in the perturbation sector.

The table below 1 displays all the modified gravity theories that can be described, at linear level, by the EFT of DE. They are classified by the couplings they require.

	$\mu = \frac{d\log M^2(t)}{dt}$	λ	$\mathcal{C}$	μ_2^2	μ_3	ϵ_4
ΛCDM	0	const.	0	0	0	0
Quintessence	0	✓	✓	0	0	0
k-essence	0	✓	✓	✓	0	0
Brans-Dicke	✓	✓	✓	0	0	0
$f(R)$	✓	✓	0	0	0	0
Kinetic braiding	0	✓	✓	✓	✓	0
DGP	✓	✓	✓	✓	✓	0
Galileon Cosmology	✓	✓	✓	✓	✓	0
$f(G)$ -Gauss-Bonnet	✓	✓	✓	✓	✓	✓
Galileons	✓	✓	✓	✓	✓	✓
Horndeski	✓	✓	✓	✓	✓	✓

Table 1: Classification of models of modified gravity by their requirement of EFT coupling functions [10].

3.2 Linear Hamiltonian stability

EFT of DE allows to assess in a comfortable way if theories are stable and physically viable. The phenomenological modelling of the couplings must be able to characterize the phase space of stable theories, namely not introducing ghosts (unbounded Hamiltonian, no ground state, related to the EFT action) nor gradient instabilities (negative speed of sound). Stability conditions can be expressed by a change of coordinates in order to make the fluctuations of scalar field reappear explicitly, namely by forcing a time diffeomorphism $t \to t + \pi(x)$ on the action (19) (Stückelberg mechanism):

$$S_\pi = \int a^3 M^2 \left[A\left(\mu, \mu_2^2, \mu_3, \epsilon_4\right)\ \dot{\pi}^2\ -\ B\left(\mu, \mu_3, \epsilon_4\right) \frac{(\vec{\nabla}\pi)^2}{a^2} \right]\ +\ \ldots\,, \tag{20}$$

where ellipsis stands for lower than quadratic derivatives terms. The stability conditions must be independently satisfied

$$A\ =\ (\mathcal{C} + 2\mu_2^2)(1+\epsilon_4) + \frac{3}{4}(\mu - \mu_3)^2 \qquad > 0 \qquad \text{no-ghost condition,} \tag{21}$$

$$B\ =\ (\mathcal{C} + \frac{\mathring{\mu}_3}{2} - \dot{H}\epsilon_4 + H\mathring{\epsilon}_4)(1+\epsilon_4) - (\mu - \mu_3)\left(\frac{\mu - \mu_3}{4(1+\epsilon_4)} - \mu - \mathring{\epsilon}_4\right) \quad \geq 0 \qquad \text{gradient-stability condition.} \tag{22}$$

The propagation speed of perturbations are defined by

$$c_s^2 = \frac{B}{A} \qquad \text{sound speed of scalar perturbations,} \tag{23}$$

$$c_T^2 = \frac{1}{1+\epsilon_4} \qquad \text{speed of gravitationnal waves.} \tag{24}$$

Please refer to appendix B to see the equations of motion of the EFToDE.

3.3 Background expansion history

The background expansion of EFT models can by entirely fixed, namely fixing the Hubble rate $H(z)$ as a function of redshift. Recently, observations tightly constrain the expansion history of the universe to that of a spatially flat ΛCDM model. In that way, one can rightfully assume

$$H^2(z) = H_0^2 \left[x_0(1+z)^3 + (1-x_0)\,(1+z)^{3(1+w_{\rm eff})} \right]\,. \tag{25}$$

The quantities x_0 –the present fractional matter density of the background– and $w_{\rm eff}$ –the effective equation of state parameter– are free parameters, though observations suggest x_0 and $w_{\rm eff}$ must be close to 0.3 and -1, respectively. Since we are interested in the recent expansion history, we have neglected the contribution of radiation.

The fractional matter density of the background reference model calculated at any epoch, x, proves a useful time variable for late-time cosmology for two reasons. It characterises the evolution of the universe by smoothly interpolating between $x = 1$, deep in the matter dominated era, and its present value $x_0 \simeq 0.3$. It has also the benefit of "zooming in" the late time epoch when the transition to dark energy domination takes place. The expression of x as a function of redshift is

$$x\ =\ \frac{x_0}{x_0 + (1-x_0)(1+z)^{3w_{\rm eff}}}\,. \tag{26}$$

The functions $M^2(t)$ (the "bare Planck mass") and $\mathcal{C}(t)$ are not an independent free functions of the formalism but are related to the non-minimal coupling μ by

$$\mu \equiv \frac{d\ln M^2(t)}{dt} \Leftrightarrow M^2(t) \;=\; \frac{M_{\rm P}^2}{(1+\epsilon_4^0)^2}\,\exp\left(\int_{t_0}^{t} dt'\mu(t')\right), \tag{27}$$

$$\mathcal{C} \;=\; \frac{1}{2}(H\mu - \dot\mu - \mu^2) - \dot H - \frac{\rho_m}{2M^2}\,. \tag{28}$$

where ρ_m represents the *physical* energy density of non-relativistic matter. By "physical" we mean the quantity appearing in the energy momentum tensor. It scales as a^{-3} since we are in the Jordan frame and $p_m \simeq 0$. With this quantity, we can define the physical fractional energy density today

$$\Omega_m^0 \equiv \frac{\rho_m(t_0)}{3M_{\rm P}^2 H_0^2}\,. \tag{29}$$

In principle, one could try to measure $\rho_m(t_0)$ by directly weighting the total amount of baryons and dark matter, for instance within a Hubble volume. It is worth emphasizing that, in theories of modified gravity, Ω_m^0 needs not be the same as x_0. The latter is a purely geometrical quantity, a proxy for the behaviour of $H(z)$. It proves useful to define

$$\kappa \;\equiv\; \frac{\Omega_m^0}{x_0}\,. \tag{30}$$

3.4 Perturbation sector

There is a range of scales on which extracting perturbation observables from modified gravity (MG) theories is relatively straightforward: the window of co-moving Fourier modes $k_{\rm sh} < k < k_{\rm nl}$. For momenta less than the non-linear scale, $k_{\rm nl} \simeq (10\,{\rm Mpc})^{-1}$, one can trust linear perturbation theory. For momenta well above the sound horizon scale $k_{\rm sh} \simeq aH/c_s$ (c_s is the speed of sound of dark energy fluctuations), one can neglect the time derivatives of the metric and scalar fluctuations in the linear equations, the so called *quasi-static approximation.* In the quasi-static regime, it is possible to compute algebraically the effective Newton constant $G_{\rm eff}(t,k)$ and the gravitational slip parameter $\eta(t,k)$ of a given modified gravity theory. The entire set of perturbation equations then reduces to

$$-\frac{k^2}{a^2}\Phi = 4\pi G_{\rm eff}(t,k)\delta\rho_m\,, \tag{31}$$

$$\eta(t,k) = \frac{\Psi}{\Phi}, \tag{32}$$

where the following convention for the perturbed metric in Newtonian gauge is adopted

$$ds^2 = -(1+2\Phi)dt^2 + a^2(1-2\Psi)\delta_{ij}dx^i dx^j\,. \tag{33}$$

The effective gravitational constant $G_{\rm eff}$ reads

$$G_{\rm eff} \;=\; \frac{1}{8\pi M(t)^2(1+\epsilon_4)^2}\;\frac{2\mathcal{C}+\mathring{\mu}_3 - 2\dot H\epsilon_4 + 2H\mathring{\epsilon}_4 + 2(\mu+\mathring{\epsilon}_4)^2 + \;Y_{\rm IR}}{2\mathcal{C}+\mathring{\mu}_3 - 2\dot H\epsilon_4 + 2H\mathring{\epsilon}_4 + 2\dfrac{(\mu+\mathring{\epsilon}_4)(\mu-\mu_3)}{1+\epsilon_4} - \dfrac{(\mu-\mu_3)^2}{2(1+\epsilon_4)^2} + \;Y_{\rm IR}}\,, \tag{34}$$

where*

$$\mathring{\mu}_3 \;\equiv\; \dot\mu_3 + \mu\mu_3 + H\mu_3, \tag{35}$$

$$\mathring{\epsilon}_4 \;\equiv\; \dot\epsilon_4 + \mu\epsilon_4 + H\epsilon_4\,, \tag{36}$$

$$Y_{\rm IR} \;\equiv\; 3\left(\frac{a}{k}\right)^2 \left[2\dot H\mathcal{C} - \dot H\mathring{\mu}_3 + \ddot H(\mu-\mu_3) - 2H\dot H\mu_3 - 2H^2(\mu^2+\dot\mu)\right]. \tag{37}$$

and the gravitational slip parameter η reads

$$\eta \;= 1 - \frac{(\mu+\mathring{\epsilon}_4)(\mu+\mu_3+2\mathring{\epsilon}_4) - \epsilon_4(2\mathcal{C}+\mathring{\mu}_3 - 2\dot H\epsilon_4 + 2H\mathring{\epsilon}_4) + \epsilon_4\cdot Y_{\rm IR}}{2\mathcal{C}+\mathring{\mu}_3 - 2\dot H\epsilon_4 + 2H\mathring{\epsilon}_4 + 2(\mu+\mathring{\epsilon}_4)^2 + Y_{\rm IR}}\,. \tag{38}$$

from these two quantities it is possible to compute other observables. For instance the growth rate is obtained by solving

*The formula for $Y_{\rm IR}$ is somewhat approximated. Its computation does not seem to have been done rigorously in literature yet.

$$3w_{\rm eff}(1-x)xf'(x)+f(x)^2+\left[2-\frac{3}{2}\left(w_{\rm eff}(1-x)+1\right)\right]f(x)=\frac{3x}{2}\kappa\frac{G_{\rm eff}}{G_{\rm N}}\,. \tag{39}$$

One can then compute the growth function $f\sigma_8$ that is more related to observations. Another observable is the lensing potential

$$\Sigma \;=\; \frac{G_{\rm eff}}{G_{\rm N}}\,\frac{1+\eta}{2}\,. \tag{40}$$

The parametrisation of the coupling functions is free up to dimensionality consistency. A simple ansatz to model late time cosmic acceleration could be

$$\mu \;=\; \eta_1\,H\,(1-x) \tag{41a}$$
$$\mu_2^2 \;=\; \eta_2\,H^2\,(1-x) \tag{41b}$$
$$\mu_3 \;=\; \eta_3\,H\,(1-x)\,, \tag{41c}$$
$$\epsilon_4 \;=\; \eta_4\,(1-x)\,, \tag{41d}$$

where η_i are dimensionless couplings taken to be constant in the simplest case.

4 Matter power spectrum updated to modified gravity

The form of the non-linear equation for matter density contrast within the context of MG is still to be fully understood. Nevertheless, a priori one could have some intuition about where MG could take place in the computation of the power spectrum. Schematically, one can decompose it as

$$P_{1-{\rm loop}}=P_{11}+P_{13}+P_{22}+P_{\rm CT}+P_*\;. \tag{42}$$

From the agreement of the assumption in both EFT's (see table below) it seems legitimate to promote $G_{\rm N}$ to $G_{\rm eff}$. In that way, the computations of P_{11}, P_{13}, P_{22}, $P_{\rm CT}$ would be rather straightforward and would result in the inclusion of an EFToDE growing mode. However, the addition of the extra scalar field π translates into new mixing/couplings between the fields (Φ, Ψ, π). Hence we should expect to have a "pure" MG contribution to the power spectrum P_*. In the following subsections we detail more the computations of each terms.

	Hypothesis	EFToLSS	EFToDE
1	Perfect fluid behaviour	$\surd^*$	$\surd$
2	ΛCDM background	$\surd$	$\surd$
3	Matter/DE flat universe ($\Omega_m+\Omega_{DE}=1$)	$\surd$	$\surd$
4	Linear order in Φ & Ψ	$\surd$	$\surd$
5	Special relativistic corrections dropped (in $T_{\mu\nu}$)	$\surd$	$\surd$
6	Quasi-static approximation	$\surd$	$\surd$
7	Absence of anisotropic stress	$\surd$	X

1 → In EFToLSS deviations from perfect fluid behaviour are encoded in counter-terms.

7 → This effect will be taken into account in the equations of motion with $G_{\rm eff}$.

4.1 Linear power spectrum

The linear part of the equation would be that of EFToDE, thus

$$\delta_k''-\frac{1-\epsilon}{1+z}\delta_k'-4\pi G_{\rm eff}(z)\frac{\rho_m(z)}{(1+z)^2}\delta_k=0\;, \tag{43}$$

or equivalently

$$\delta_k''-\frac{1-\epsilon}{1+z}\delta_k'-\frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\,H_0^2\,\Omega_{m,0}\,(1+z)\,\delta_k=0\;. \tag{44}$$

At late times and for sub-horizon modes, δ_k grows according to the linear contribution in equation (43) and can be decomposed as

$$\delta^{(1)}(k,z)=A_k\mathcal{D}_{\rm MG}(z)\;, \tag{45}$$

where A_k is an amplitude factor that encodes the dependence on k, and $\mathcal{D}_{\rm MG}(z)$ is the linear growing mode of the EFT model. In analogy to EFToLSS, $A_k\equiv\delta_k^*=T_k(z^*)\Phi^{prim}$. To compute $\mathcal{D}_{\rm MG}(z)$ one uses "Green's method". Therefore, it is given by solving

$$\frac{\mathrm{d}^2\mathcal{D}_{\rm MG}(\mathrm{z})}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{d}\mathcal{D}_{\rm MG}(\mathrm{z})}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ \mathcal{D}_{\rm MG}(z) = 0\ , \tag{46}$$

where the initial conditions should be fixed to be the one of ΛCDM at early times (redshift z_*), *i.e* deep in matter domination. Another possibility would be

$$\mathcal{D}_{\rm MG}(z) = \mathcal{N}\ e^{-\int_0^{z_{dec}} \frac{f^{\Lambda\rm CDM}}{1+\tilde{z}}\mathrm{d}\tilde{z}}\ e^{-\int_{z_{dec}}^{z} \frac{f^{\rm eft}}{1+\tilde{z}}\mathrm{d}\tilde{z}}\ , \tag{47}$$

where $\mathcal{N}$ a normalisation factor computed according to the initial conditions and f is the growth rate (using (39)).

Using the definition of power spectrum and the linear solution for the density contrast

$$(2\pi)^2\delta(\vec{k}+\vec{k}')P_{11}(k,z) = \langle\delta^{(1)}_{\mathbf{k}}\delta^{(1)}_{\mathbf{k}'}\rangle = \mathcal{D}^2_{\rm MG}(z)\langle\delta^*_k\delta^*_{k'}\rangle. \tag{48}$$

and that of the primordial power spectrum $\langle\delta^*_k\delta^*_{k'}\rangle = (2\pi)^2\delta(\vec{k}+\vec{k}')\mathcal{P}_R(k,z)$, the linear power spectrum reads

$$P_{11}(k,z) = \mathcal{D}^2_{\rm MG}(z)\mathcal{P}_R(k,z). \tag{49}$$

4.2 Quadratic contribution

The quadratic power spectrum reads

$$\begin{aligned} P_{22}(k,z) = 2\int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})\mathcal{P}_R(\vec{k}-\vec{q})[\alpha(\vec{q},\vec{k}-\vec{q})A(z) + \gamma(\vec{q},\vec{k}-\vec{q})B(z)] \\ \times[\alpha(-\vec{q},\vec{q}-\vec{k})A(z) + \gamma(-\vec{q},\vec{q}-\vec{k})B(z)]\ . \end{aligned} \tag{50}$$

Where A and B functions satisfy equations

$$\frac{\mathrm{d}^2\mathrm{A(z)}}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{dA(z)}}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ A(z) = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ \mathcal{D}^2_{\rm MG}(z), \quad (i.c:\ A(z=z_*) = A'(z=z_*) = 0) \tag{51}$$

$$\frac{\mathrm{d}^2\mathrm{B(z)}}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{dB(z)}}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ B(z) = \mathcal{D}'^{\,2}_{\rm MG}(z), \quad (i.c:\ B(z=z_*) = B'(z=z_*) = 0) \tag{52}$$

4.3 Cubic contribution

The cubic contribution is

$$\begin{aligned} P_{13}(k,z) = 4\,\mathcal{D}_{\rm MG}(z)\mathcal{P}_R(k)\int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})[\alpha(\vec{k},-\vec{q})[\alpha(\vec{k}-\vec{q},\vec{q})F(z) + \gamma(\vec{k}-\vec{q},\vec{q})D(z)] \\ +\gamma(\vec{k},-\vec{q})[\alpha(\vec{k}-\vec{q},\vec{q})E(z) + \gamma(\vec{k}-\vec{q},\vec{q})G(z)]] \end{aligned} \tag{53}$$

where F and G functions are obtained from

$$\frac{\mathrm{d}^2\mathrm{F(z)}}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{dF(z)}}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ F(z) = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ \mathcal{D}_{\rm MG}(z)\ A(z), \quad (i.c:\ F(z=z_*) = F'(z=z_*) = 0) \tag{54}$$

$$\frac{\mathrm{d}^2\mathrm{G(z)}}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{dG(z)}}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ G(z) = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ \mathcal{D}_{\rm MG}(z)\ B(z), \quad (i.c:\ G(z=z_*) = G'(z=z_*) = 0) \tag{55}$$

D and E functions

$$\frac{\mathrm{d}^2\mathrm{D(z)}}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{dD(z)}}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ D(z) = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ \mathcal{D}'_{\rm MG}(z)\ A'(z), \quad (i.c:\ D(z=z_*) = D'(z=z_*) = 0) \tag{56}$$

$$\frac{\mathrm{d}^2\mathrm{E(z)}}{\mathrm{dz}^2} - \frac{1-\epsilon}{1+z}\frac{\mathrm{dE(z)}}{\mathrm{dz}} - \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ E(z) = \frac{3}{2}\frac{G_{\rm eff}}{G_{\rm N}}(z)\ H_0^2\ \Omega_{m,0}\ (1+z)\ \mathcal{D}'_{\rm MG}(z)\ B'(z), \quad (i.c:\ E(z=z_*) = E'(z=z_*) = 0) \tag{57}$$

4.4 Counter-term

Counter-term contribution will now read

$$P_{\rm CT}(k,z) = c_{\rm CT}^{\rm MG}(z)^2 \frac{k^2}{a^2H^2}\mathcal{D}_{\rm MG}(z)\mathcal{P}_R(k) \tag{58}$$

where

$$c_{\rm CT}^{\rm MG}(z)^2 = \int {\rm d}\tilde{z}\mathcal{G}(z,\tilde{z})\frac{Z_\delta(\tilde{z})\mathcal{D}_{\rm MG}(\tilde{z}) + Z_\Theta(\tilde{z})H(1+\tilde{z})\mathcal{D}'_{\rm MG}(\tilde{z})}{(1+\tilde{z})^2}, \tag{59}$$

being $\mathcal{G}(z,\tilde{z})$ the Green's function.

The expression for (59) would not need to be computed since $P_{\rm CT}$ will absorb the UV divergences in P_{13}. To understand how counter-terms work, let's focus exclusively on the analytic part of the power spectrum, i.e. in the high-energy regime where a Taylor expansion around $k = 0$ is allowed. In this limit, we Taylor expand P_{13}. That means P_{13} and $P_{\rm CT}$ show the same k dependence, that is, k^2. In order to make sure that the UV divergences are absorbed by the counter-terms, $c_{\rm CT^2}^{\rm MG}\mathcal{D}_{\rm MG}(z)/a^2H^2$ must tend to the same z-evolution as the cubic contribution, $\alpha(z)$, up to a constant factor, $\mathcal{C}_{\rm CT}$. The procedure is analogous to that presented in appendix A, equations (95) and (96). Therefore, by inspection of the low-k behaviour

$$\begin{aligned} P_{13}(k,z) + P_{\rm CT}(k,z) \simeq& \mathcal{D}_{\rm MG}(z)\mathcal{P}_R(k)[\mathcal{A}(\Lambda)\alpha(z)]k^2 + \mathcal{D}_{\rm MG}(z)\mathcal{P}_R(k)[\mathcal{C}_{\rm CT}\alpha(z)]k^2 \\ =& \mathcal{D}_{\rm MG}(z)\mathcal{P}_R(k)[\mu\alpha(z)]k^2 \end{aligned} \tag{60}$$

μ being the re-normalisation parameter that should be fitted at early times by matching to ΛCDM power spectrum and,

$$\mathcal{A}(\Lambda)\times\alpha(z) = \int_0^\Lambda \frac{{\rm d}q}{60\pi^2}\mathcal{P}_R(q)\times[7F(z) - 18D(z) - 28E(z) + 2G(z)]\,. \tag{61}$$

4.5 Divergences and non-linear effects

We have seen how counter-terms work in the limit $k \to 0$. However, it is important to describe the whole picture at any scale $k_{sh} < k < k_{nl}$ (and arbitrary redshift within matter dominated era) where the power spectrum presents both local and non local effects.

So far, we do not need to worry about P_{22} since its UV divergences are absorbed at 2-loops by the stochastic term. Therefore, one needs to split the P_{13} integral in two: the analytic part (cut-off dependent) and the non analytic part (low k regime). If MG is added, theoretically we would expect (53) to manifest those effects in the manner explained below. Equation (53) can be rewritten as

$$P_{13}(k,z) = \mathcal{D}_{\rm MG}(z)\mathcal{P}_R(k)\sum_i \mathcal{I}_i(\Lambda)\mathcal{F}_i(z), \tag{62}$$

then

$$\begin{aligned} \mathcal{I}_i(\Lambda) =& \int^\Lambda \frac{{\rm d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})K_i(\vec{k},-\vec{q}) \\ =& \underbrace{\int_0^{k*} \frac{{\rm d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})K_i(\vec{k},-\vec{q})}_{\text{Non-analytic}\propto f(k)} + \underbrace{\int_{k*}^\Lambda \frac{{\rm d}^3\vec{q}}{(2\pi)^3}\mathcal{P}_R(\vec{q})K_i(\vec{k},-\vec{q})}_{\text{Analytic}\propto k^2} \\ =& a_i^{\rm MG}(\Lambda)\cdot k^2 + b_i^{\rm MG}\cdot f(k) + O(k^4). \end{aligned} \tag{63}$$

A priori, $f(k)$ could be any function of k, namely $\log(k)$, $\cos(k)$, fractional power of k^2, etc. In opposition to the GR case in EFToLSS that is thought to be k^3 [3]. Note that from such discrepancies in k-behaviour from the non-analytic part between GR and MG one should be able to extract part of information on screening (further information would come from the new coupling terms). Therefore, we would be interested in identifying $a_i^{\rm MG}(\Lambda)$ (re-normalisation) and $b_i^{\rm MG}$ (predicted by the theory). In order to do that, we present schematically the steps to follow

1. We would need to fix the value of redshift, say $z = 0$, and the cut-off $\Lambda \equiv$ const.
   (i) To compute (53) numerically.
   (ii) To compare with the theoretical expression
   $$P_{13}(k,z) = \mathcal{D}_{\rm MG}(z)\mathcal{P}_R(k)[a^{\rm MG}(\Lambda)\cdot k^2 + b^{\rm MG}\cdot f(k) + O(k^4)]. \tag{64}$$
   (iii) We would probably need an ansatz for $f(k)$.

(iv) To adjust the theoretical curve (64) from to the numerical output (53). Doing so we could obtain $\{a^{\rm MG}(\Lambda \equiv {\rm const}), b^{\rm MG}\}$.

2. Repeat for several values of the cut-off to get $\{a^{\rm MG} \equiv a^{\rm MG}(\Lambda), b^{\rm MG} \equiv {\rm const}\}$.

We might want to focus on IR divergences at some point in our analysis. According to [3], $P_{13} + P_{22}$ is free of IR divergences, whereas each term separately is not. In our new approach (EFToLSS+MG), it is expected to be true but it needs to be checked. That means we might want to perform the above procedure not only for P_{13} but for $P_{13} + P_{22}$.

4.6 Pure MG contributions

As we have said EFToDE encompasses alternative theories to the concordance cosmological model with an extra scalar field, i.e. the Goldstone boson of spontaneously broken time translations π. Therefore, in addition to the results of the direct promotion of $G_{\rm N}$ to $G_{\rm eff}$, it is expected to get new contributions in the RHS of the perturbation equation that will lead to add P_*. Those contributions might depend on the frame used, namely:

- In the Jordan frame the new couplings which appear are $\delta\Phi$, $\pi\Phi$ and $\pi\Psi$ according to the EFToDE action written in Newtonian gauge
$$S = \int a\, M^2 \left[(\vec\nabla\Psi)^2 - 2(1+\epsilon_4)\vec\nabla\Phi\vec\nabla\Psi - 2(\mu+\mathring{\epsilon}_4)\vec\nabla\Psi\vec\nabla\pi + (\mu-\mu_3)\vec\nabla\Phi\vec\nabla\pi - \left(\mathcal{C} + \frac{\mathring{\mu}_3}{2} - \dot{H}\epsilon_4 + H\mathring{\epsilon}_4\right)(\vec\nabla\pi)^2\right] - a^3\,\Phi\delta\rho_m, \tag{65}$$
where $\delta\rho_m$ is the perturbation of the non-relativistic energy density, a dot means derivative w.r.t. proper time and π represents the perturbation of the scalar field. Therefore, one could expect terms of the form
$$\sim \int \frac{{\rm d}^3\vec q{\rm d}^3\vec r}{(2\pi)^6} K_1(\vec k,\vec q,\vec r;z)\delta\Phi, \tag{66}$$
$$\sim \int \frac{{\rm d}^3\vec q{\rm d}^3\vec r}{(2\pi)^6} K_2(\vec k,\vec q,\vec r;z)\pi\Phi, \tag{67}$$
$$\sim \int \frac{{\rm d}^3\vec q{\rm d}^3\vec r}{(2\pi)^6} K_3(\vec k,\vec q,\vec r;z)\pi\Psi. \tag{68}$$

- In the Einstein frame matter is directly coupled to the propagating degree of freedom, that is, $\delta\pi$ and so one could expect a new contribution of the form
$$\sim \int \frac{{\rm d}^3\vec q{\rm d}^3\vec r}{(2\pi)^6} K_4(\vec k,\vec q,\vec r;z)\delta\pi. \tag{69}$$

Let us stick to the Jordan frame since it is the one we are used to, in summary the non-linear matter density contrast equation at 1-loop would therefore read

$$\begin{aligned}
\delta_k'' - \frac{1-\epsilon}{1+z}\delta_k' - 4\pi G_{\rm eff}(t)\frac{\rho_m(z)}{(1+z)^2}\delta_k = \;& \frac{k^2}{H^2a^2}\frac{Z_\delta\delta_k + Z_\Theta H(1+z)\delta_k'}{(1+z)^2} \\
&+ \int \frac{{\rm d}^3\vec q{\rm d}^3\vec r}{(2\pi)^6}(2\pi)^3\delta(\vec k-\vec q-\vec r)\left[\alpha(\vec q,\vec r)\frac{3}{2}\frac{\Omega_M(z)}{(1+z)^2}\delta(\vec q)\delta(\vec r) + \gamma(\vec q,\vec r)\delta'(\vec q)\delta'(\vec r)\right] \\
&+ \int \frac{{\rm d}^3\vec q{\rm d}^3\vec r}{(2\pi)^6} K_1(\vec k,\vec q,\vec r;z)\delta\Phi \\
&+ \int \frac{{\rm d}^3\vec q{\rm d}^3\vec r}{(2\pi)^6} K_2(\vec k,\vec q,\vec r;z)\pi\Phi \\
&+ \int \frac{{\rm d}^3\vec q{\rm d}^3\vec r}{(2\pi)^6} K_3(\vec k,\vec q,\vec r;z)\pi\Psi.
\end{aligned} \tag{70}$$

We should think of the new type of diagrams that these new terms imply. Hence the first big algebraic task of the project will be to compute these pure MG contributions. It is worth thinking about it before and try to estimate how much they contribute, they might be negligible. The other terms (P_{ii}) will only require (tedious) coding efforts.

5 Prospects

Let us establish "cahier des charges" of the project:

1. Carry on understanding what we are doing,
   - Be sure about the matching of the assumptions of both formalism.
   - Think whether the neglected 3-velocity potential in Poisson equation of EFToDE can be a problem for the EFToLSS description.

- Spot correctly where all the MG effects arise in the computations in opposition to the standard case, in particular in the purely MG contribution of the RHS of the δ_k equation (new diagrams involving π).
- See if there are problems between the two EFTs on IR scales.
- Be careful about initial conditions in an MG scenario (primordial power spectrum when $\eta \neq 1$)
- Think about the use of time variables (especially for coding). EFToLSS is presented for z but analytical computations in EFToDE are simpler with x and has advantages to model late time acceleration.

2. Check whether the re-normalisation of divergences needs to be done for P_{13} or $P_{13} + P_{22}$ in order to be IR safe.
3. Think about the need of computing the 2-loop corrections.
4. Compute the power spectrum in a EFT model (algebra and code).
5. Compare the power spectrum for different EFT models.
6. Look at the impacts on observables (f, σ_8, etc.).
7. Extract information on screening.
   - How should we extract and parametrise it? An idea would be to relate it to a $G_{\text{eff,sc}}(t, k)$, propose its parametrisation and do a parameter fitting process to find the parametrisation parameters.
   - Be careful and think about how to distinguish between genuine MG/screening effects and unknown structure formation effects in GR.

A Perturbation sector within EFToLSS

A.1 Density contrast

1. **Linear solution**
At late times, inside the horizon, δ_k grows according to the linear contribution in equation (6)
$$\delta^{(1)}(k,z) = g(z)\delta_k^*. \tag{71}$$
To get δ_k^* from primordial initial condition we need a transfer function. This needs the details of both matter and radiation perturbations in order to capture the evolution of the gravitational potential, Φ, during matter and radiation domination. Therefore, by isotropy argument, all modes with the same k will evolve in exactly same way, thus
$$\delta_k^* \equiv T_k(z_*)\Phi_k^{Primordial} \tag{72}$$
where $T_k(z)$ is a transfer function.

2. **Counter-term contribution**
$$\delta^{CT}(k,z) = \frac{k^2}{H^2a^2}c_{CT}^2(z)\delta_k^* \tag{73}$$
where
$$c_{CT}^2(z) = \int \mathrm{d}\tilde{z}\mathcal{G}(\tilde{z},z)\frac{1}{(1+\tilde{z})^2}\left[Z_\delta(\tilde{z})g(\tilde{z}) + Z_\Theta(\tilde{z})H(1+\tilde{z})g'(\tilde{z})\right], \tag{74}$$
being $\mathcal{G}(\tilde{z},z)$ the Green's function.

3. **Quadratic solution**
From quadratic terms in equation (6)
$$\delta^{(2)}(k,z) \supseteq \int \frac{\mathrm{d}^3\vec{q}\mathrm{d}^3\vec{r}}{(2\pi)^6}(2\pi)^3\delta(\vec{k}-\vec{q}-\vec{r})[\alpha(\vec{q},\vec{r})A(z)+\gamma(\vec{q},\vec{r})B(z)]\delta_{\vec{q}}^*\delta_{\vec{r}}^* \tag{75}$$
where
$$A(z) = \int_{z_*}^{z}\mathrm{d}\tilde{\mathrm{z}}\mathcal{G}(\tilde{\mathrm{z}},\mathrm{z})\frac{3}{2}\frac{\Omega_\mathrm{M}(\tilde{\mathrm{z}})}{(1+\tilde{\mathrm{z}})^2}\mathrm{g}(\tilde{\mathrm{z}})^2 \tag{76}$$
and
$$B(z) = \int_{z_*}^{z}\mathrm{d}\tilde{\mathrm{z}}\mathcal{G}(\tilde{\mathrm{z}},\mathrm{z})\mathrm{g}'(\tilde{\mathrm{z}})^2. \tag{77}$$

4. **Qubic solution**
$$\begin{aligned}\delta^{(3)}(k,z) \supseteq 2\int \frac{\mathrm{d}^3\vec{q}\mathrm{d}^3\vec{s}\mathrm{d}^3\tilde{\mathrm{r}}}{(2\pi)^9}(2\pi)^3\delta(\vec{k}-\vec{q}-\vec{s}-\vec{r})[\alpha(\vec{s},\vec{r})[\alpha(\vec{s}+\vec{r},\vec{q})F(z)+\gamma(\vec{s}+\vec{r},\vec{q})D(z)]\\ +\gamma(\vec{s},\vec{r})[\alpha(\vec{s}+\vec{r},\vec{q})G(z)+\gamma(\vec{s}+\vec{r},\vec{q})E(z)]]\delta_{\vec{q}}^*\delta_{\vec{s}}^*\delta_{\vec{r}}^*.\end{aligned} \tag{78}$$
The coefficients are given by
$$F(z) = \int_{z*}^{z}\mathrm{d}\tilde{\mathrm{z}}\mathcal{G}(\tilde{\mathrm{z}},\mathrm{z})\frac{3}{2}\frac{\Omega_\mathrm{M}(\tilde{\mathrm{z}})}{(1+\tilde{\mathrm{z}})^2}\mathrm{g}(\tilde{\mathrm{z}})\mathrm{A}(\tilde{\mathrm{z}}) \tag{79}$$
$$G(z) = \int_{z*}^{z}\mathrm{d}\tilde{\mathrm{z}}\mathcal{G}(\tilde{\mathrm{z}},\mathrm{z})\frac{3}{2}\frac{\Omega_\mathrm{M}(\tilde{\mathrm{z}})}{(1+\tilde{\mathrm{z}})^2}\mathrm{g}(\tilde{\mathrm{z}})\mathrm{B}(\tilde{\mathrm{z}}) \tag{80}$$
$$D(z) = \int_{z*}^{z}\mathrm{d}\tilde{\mathrm{z}}\mathcal{G}(\tilde{\mathrm{z}},\mathrm{z})\mathrm{g}'(\tilde{\mathrm{z}})\mathrm{A}'(\tilde{\mathrm{z}}) \tag{81}$$
$$E(z) = \int_{z*}^{z}\mathrm{d}\tilde{\mathrm{z}}\mathcal{G}(\tilde{\mathrm{z}},\mathrm{z})\mathrm{g}'(\tilde{\mathrm{z}})\mathrm{B}'(\tilde{\mathrm{z}}) \tag{82}$$
where
$$A'(z) = \int_{z*}^{z}\mathrm{d}\tilde{\mathrm{z}}\frac{\partial\mathcal{G}(\tilde{\mathrm{z}},\mathrm{z})}{\partial\mathrm{z}}\frac{3}{2}\Omega_\mathrm{M}(\tilde{\mathrm{z}})\frac{\mathrm{g}(\tilde{\mathrm{z}})^2}{(1+\tilde{\mathrm{z}})^2} \tag{83}$$

and

$$B'(z) = \int_{z*}^{z} \mathrm{d}\tilde{z} \frac{\partial \mathcal{G}(\tilde{z}, z)}{\partial z} g'(\tilde{z})^2 \tag{84}$$

since $G(z, z) = 0$ for a retarded (advanced) Green's function.

A.2 Matter power spectrum

From equation (11), the definition of linear power spectrum, quadratic, cubic and counter-term contributions would be

$$< \delta^{(1)}(k, z)\delta^{(1)}(k', z) >= (2\pi)^3 \delta_D(\vec{k} + \vec{k}')P_{11}(k, z), \tag{85}$$

$$< \delta^{(2)}(k, z)\delta^{(2)}(k', z) >= (2\pi)^3 \delta_D(\vec{k} + \vec{k}')P_{22}(k, z), \tag{86}$$

$$< \delta^{(1)}(k, z)\delta^{(3)}(k', z) >= (2\pi)^3 \delta_D(\vec{k} + \vec{k}')P_{13}(k, z), \tag{87}$$

and

$$< \delta^{(1)}(k, z)\delta^{CT}(k', z) >= (2\pi)^3 \delta_D(\vec{k} + \vec{k}')P_{CT}(k, z). \tag{88}$$

Computations yield the following expressions:

1. **Linear power spectrum**

$$P_{11}(k, z) = g(z)^2 \mathcal{P}_R(k) \tag{89}$$

2. **Counter-term contribution**

$$P_{CT}(k, z) = c_{CT}(z)^2 \frac{k^2}{a^2 H^2} g(z)\mathcal{P}_R(k) \tag{90}$$

3. **Quadratic power spectrum**

$$\begin{aligned} P_{22}(k, z) = 2 \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3} \mathcal{P}_R(\vec{q})\mathcal{P}_R(\vec{k} - \vec{q})[\alpha(\vec{q}, \vec{k} - \vec{q})A(z) + \gamma(\vec{q}, \vec{k} - \vec{q})B(z)] \\ \times[\alpha(-\vec{q}, \vec{q} - \vec{k})A(z) + \gamma(-\vec{q}, \vec{q} - \vec{k})B(z)] \end{aligned} \tag{91}$$

4. **Qubic contribution**

$$\begin{aligned} P_{13}(k, z) = 4g(z)\mathcal{P}_R(k) \int \frac{\mathrm{d}^3\vec{q}}{(2\pi)^3} \mathcal{P}_R(\vec{q})[\alpha(\vec{k}, -\vec{q})[\alpha(\vec{k} - \vec{q}, \vec{q})F(z) + \gamma(\vec{k} - \vec{q}, \vec{q})D(z)] \\ +\gamma(\vec{k}, -\vec{q})[\alpha(\vec{k} - \vec{q}, \vec{q})E(z) + \gamma(\vec{k} - \vec{q}, \vec{q})G(z)]] \end{aligned} \tag{92}$$

At low-k behaviour,

$$P_{13}(k, z) \simeq g(z)\mathcal{P}_R(k)(\alpha_2(z) \times A_2(\Lambda))k^2, \tag{93}$$

where

$$\alpha_2(z) \times A_2(\Lambda) = \underbrace{[7F(z) - 18D(z) - 28E(z) + 2G(z)]}_{\text{finite}} \times \underbrace{\int_0^\Lambda \frac{\mathrm{d}q}{60\pi^2} \mathcal{P}_R(q)}_{\text{UV divergent}}. \tag{94}$$

The 1-loop power spectrum prediction would be the sum of P_{11} (89), P_{13} (93) and P_{CT} (90):

$$P_{1-loop}(k, z) = g(z)^2 \mathcal{P}_R(k) \left[1 + k^2 \left(\frac{\alpha_2(z)}{g(z)} \times A_2(\Lambda) + \frac{c_{CT}(z)^2}{a^2 H^2 g(z)}\right)\right]. \tag{95}$$

In the UV limit, terms in round bracket have no cut-off dependence since the counter-terms absorb the divergence. That means, in that limit, the factor coming from the counter-term has the same z-evolution as $\Xi(z) \equiv \alpha_2(z)/g(z)$ up to a constant. Therefore, the previous expression can be rewritten as

$$P_{1-loop}(k, z) = g(z)^2 \mathcal{P}_R(k) \left[1 + \mu\Xi(z)k^2\right] \tag{96}$$

B Equations of motion of EFToDE

- **00-component**

$$M^2\left(\frac{k^2}{a^2}\left((\mu-\mu_3)\pi-2\Psi(\epsilon_4+1)\right)+\Phi\left(2\mathcal{C}-6H^2\epsilon_4-6H^2-6H\mu+6H\mu_3+4\mu_2^2\right)+\dot{\pi}\left(-2\mathcal{C}+3H\mu-3H\mu_3-4\mu_2^2\right)\right.$$
$$\left.+3\pi\left(H\left(2\mathcal{C}-H\mu+\mu^2+\dot{\mu}\right)+\dot{H}\left(-2H\epsilon_4-\mu+\mu_3\right)\right)-3\dot{\Psi}\left(2H\epsilon_4+2H+\mu-\mu_3\right)\right)=\delta\rho_m \tag{97}$$

- **0i-component**

$$M^2\left(\pi\left(-2\mathcal{C}+H\mu+2\dot{H}\epsilon_4-\mu^2-\dot{\mu}\right)+\Phi\left(2H\epsilon_4+2H+\mu-\mu_3\right)+(\mu_3-\mu)\dot{\pi}+2\dot{\Psi}(\epsilon_4+1)\right)=-(p_m+\rho_m)v \tag{98}$$

where v is the 3-velocity potential.

- **ij-trace component**

$$M^2\left(\frac{k^2}{a^2}\left(-\frac{2}{3}\pi\left(\epsilon_4(H+\mu)+\mu+\dot{\epsilon_4}\right)+\frac{2\Psi}{3}-\frac{2}{3}\Phi(\epsilon_4+1)\right)\right.$$
$$+\Phi\left(2\mathcal{C}+6H^2\epsilon_4+6H^2+4H\mu-3H\mu_3+2H\mu\epsilon_4+2H\dot{\epsilon_4}+2\dot{H}(\epsilon_4+2)+2\mu^2+2\dot{\mu}+\dot{\mu_3}+\mu\mu_3\right)$$
$$+\pi\left(-2\mathcal{C}\mu-2\dot{\mathcal{C}}+3H^2\mu-6\mathcal{C}H-2H\mu^2-2H\dot{\mu}+\dot{H}\left(2\epsilon_4(3H+\mu)+\mu+2\dot{\epsilon_4}\right)+2\epsilon_4\ddot{H}-\mu^3-\ddot{\mu}-3\dot{\mu}\mu\right)$$
$$+\dot{\pi}\left(-2\mathcal{C}-2H\mu+3H\mu_3+2\dot{H}\epsilon_4-2\left(\mu^2+\dot{\mu}\right)-\dot{\mu_3}-\mu\mu_3\right)$$
$$\left.+\dot{\Phi}\left(2H\epsilon_4+2H+\mu-\mu_3\right)+2\dot{\Psi}\left((\epsilon_4+1)(3H+\mu)+\dot{\epsilon_4}\right)+(\mu_3-\mu)\ddot{\pi}+2(\epsilon_4+1)\ddot{\Psi}\right)=\delta p_m \tag{99}$$

- **ij-traceless component**

$$M^2\left(\pi\left(\epsilon_4(H+\mu)+\mu+\dot{\epsilon_4}\right)-\Psi+\Phi(\epsilon_4+1)\right)=\sigma \tag{100}$$

where σ is the scalar component of the anisotropic stress.

By combining eqs. (97) and (98) we obtain the relativistic generalisation of the Poisson equation :

- **Generalized Poisson equation**

$$M^2\left(\frac{k^2}{a^2}\left((\mu-\mu_3)\pi-2\Psi(\epsilon_4+1)\right)-2\dot{\pi}\left(\mathcal{C}+2\mu_2^2\right)+\Phi\left(2\mathcal{C}-3H\mu+3H\mu_3+4\mu_2^2\right)+3\dot{H}(\mu_3-\mu)\pi+(3\mu_3-3\mu)\dot{\Psi}\right)$$
$$=\delta\rho_m-3H(p_m+\rho_m)v=\rho_m\Delta_m \tag{101}$$

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