COSMOLOGICAL PERTURBATIONS
and
EFFECTIVE FIELD THEORY OF LARGE SCALE STRUCTURE

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OUTLINE...

PART I. **Standard Perturbation Theory** and **motivation**

PART II. **Effective Field Theory of Large Scale structure** in **Real Space**

PART III. **Effective Field Theory of Large Scale structure** in **Redshift Space**

PART IV. **Recap**, **conclusions** and **prospects**

PART I...

Standard Perturbation Theory and **motivation**

ABC...

1. The Study of the Universe
2. The History of the Universe
3. Observations in Cosmology
4. Concept of Redshift
5. The expansion of the Universe
6. The Power Spectrum

STANDARD PERTURBATION THEORY

1. Descriptions of observations
2. Fluid equations
3. The matter density equation
4. 2-Point correlation functions
5. 1-loop matter power spectrum

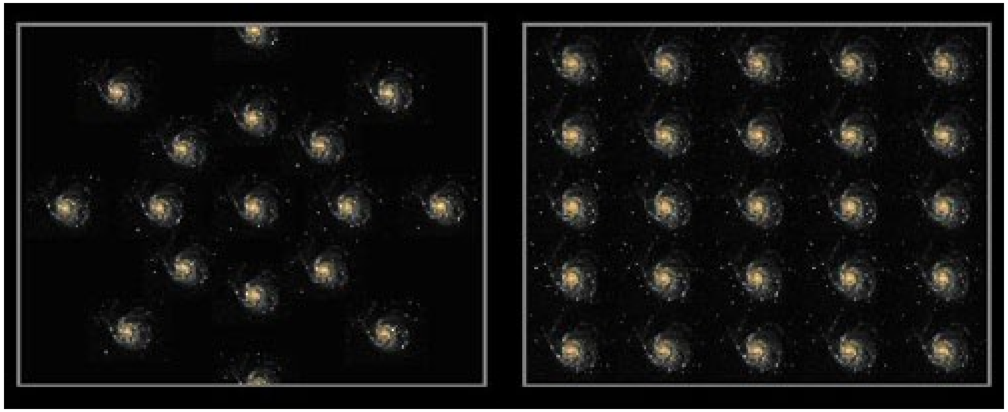
MOTIVATION FOR EXTENDING THE THEORY

ABC...

The study of the Universe

And the distribution of matter throughout the Universe

Isotropic (rotational invariance)



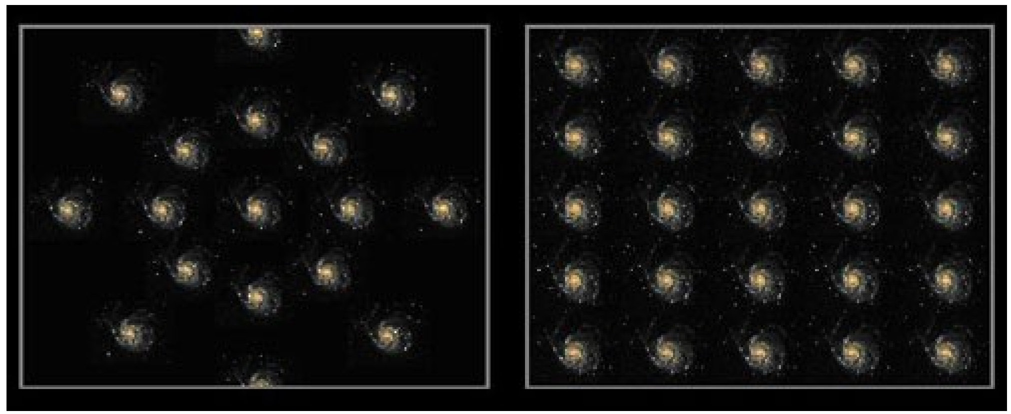
Homogeneous (translational invariance)

Cosmological principle

The study of the Universe

And the distribution of matter throughout the Universe

Isotropic (rotational invariance)



Homogeneous (translational invariance)

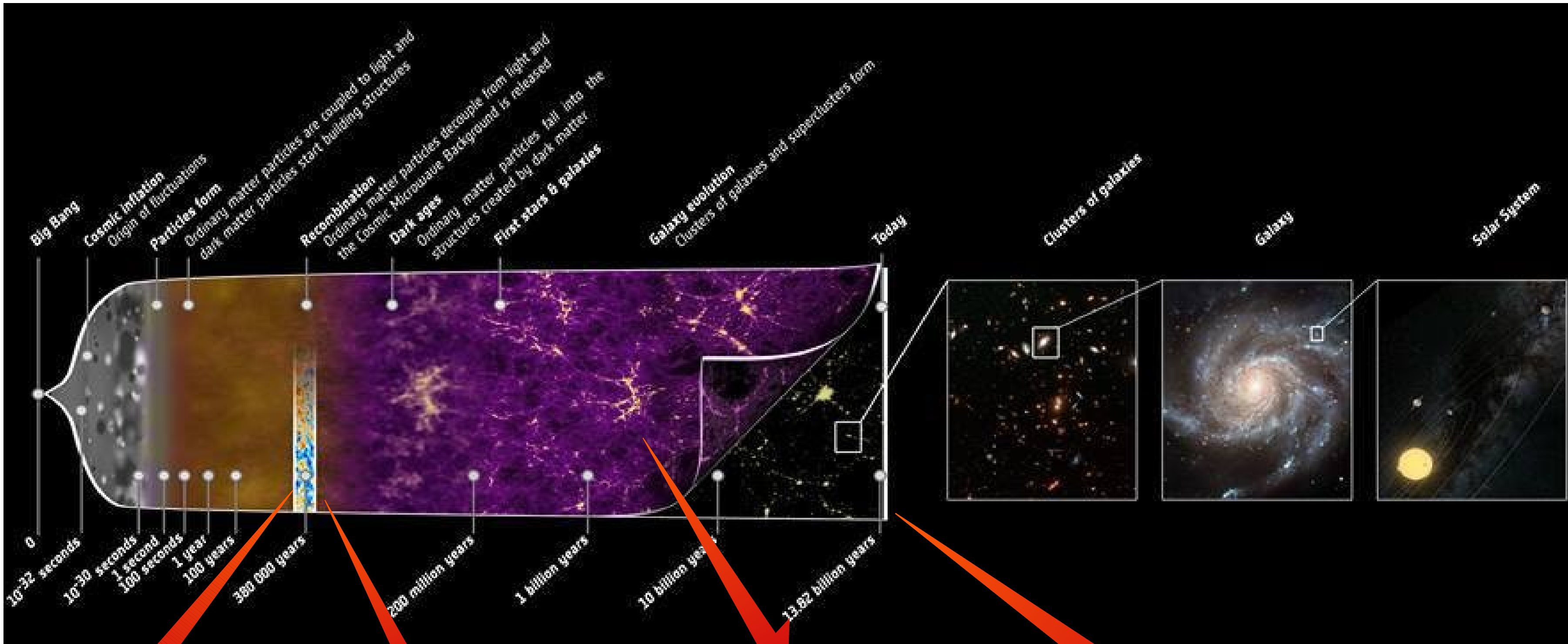
Cosmological principle

Real Universe

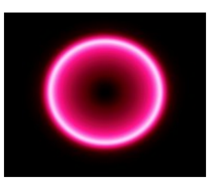
Small asymmetries and fluctuations risen from the beginning of time

STRUCTURES today!

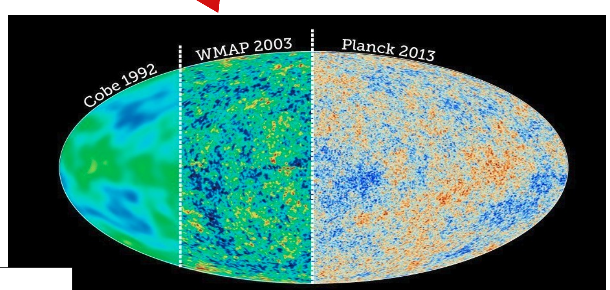
The history of our Universe

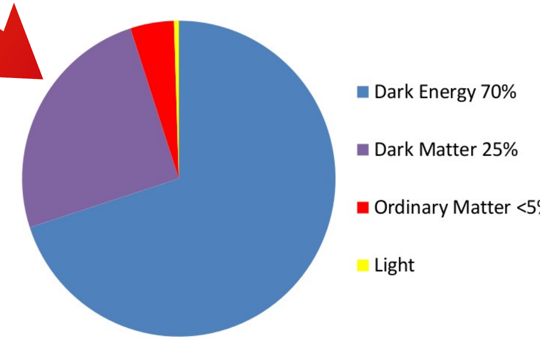


BAO

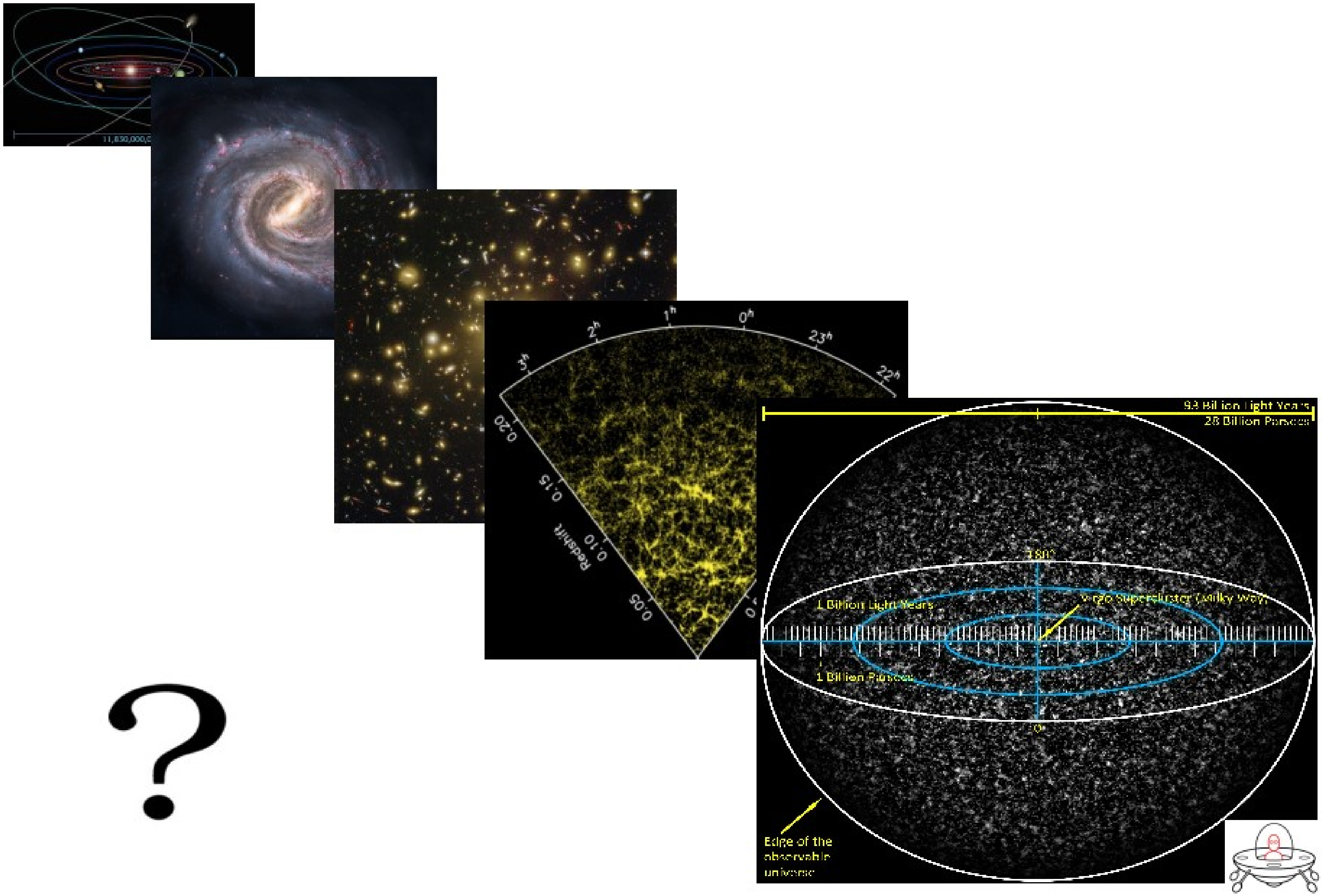


Standard Ruler





Observations in Cosmology

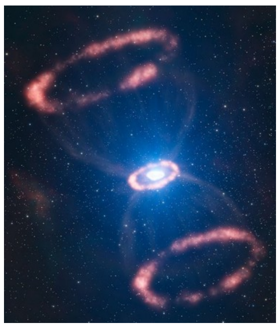


?

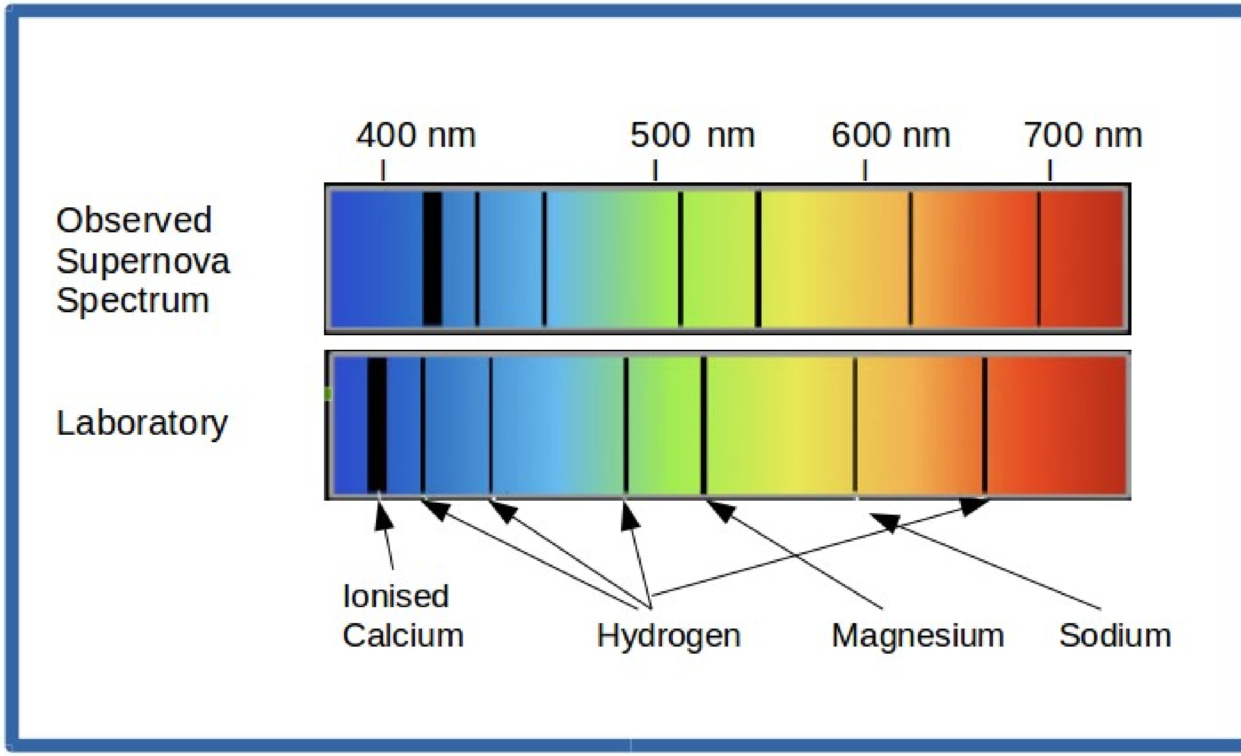
Concept of Redshift

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$

Supernovae Ia



STANDARD CANDLES



REDSHIFT

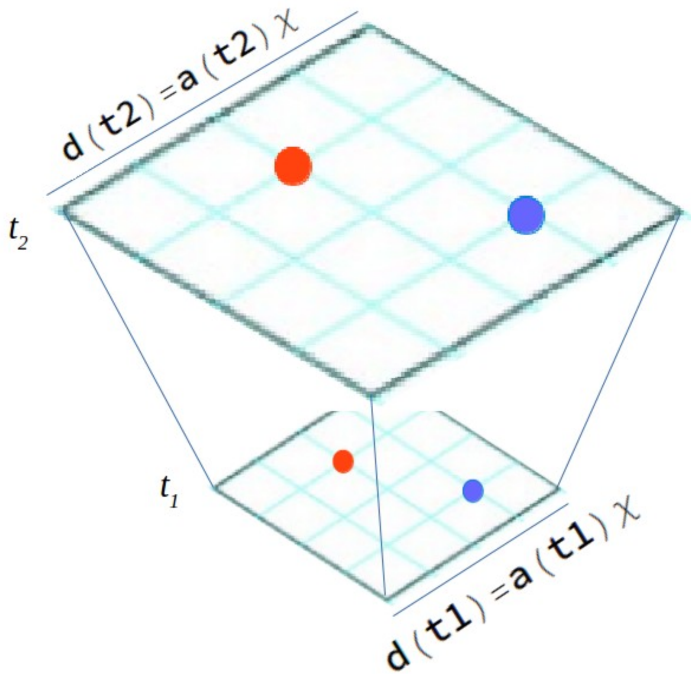
Time → *z=0 “present time”*
z> 0 past

Distances → *The farther, the older.*

Overall red-shifting of galaxy spectra?

THE UNIVERSE IS EXPANDING!

The expansion of the Universe



1. Co-moving coordinate system (grid)
Co-moving distance, χ.

2. Physical distances $d(t) = a(t)\chi$

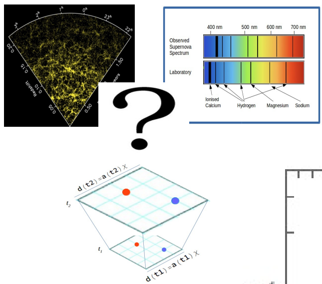
Scale factor, *a(t)*: $a(t) = \frac{1}{1+z}$

Expansion rate or **Hubble parameter:** $H(t) = \frac{\dot{a}}{a}$

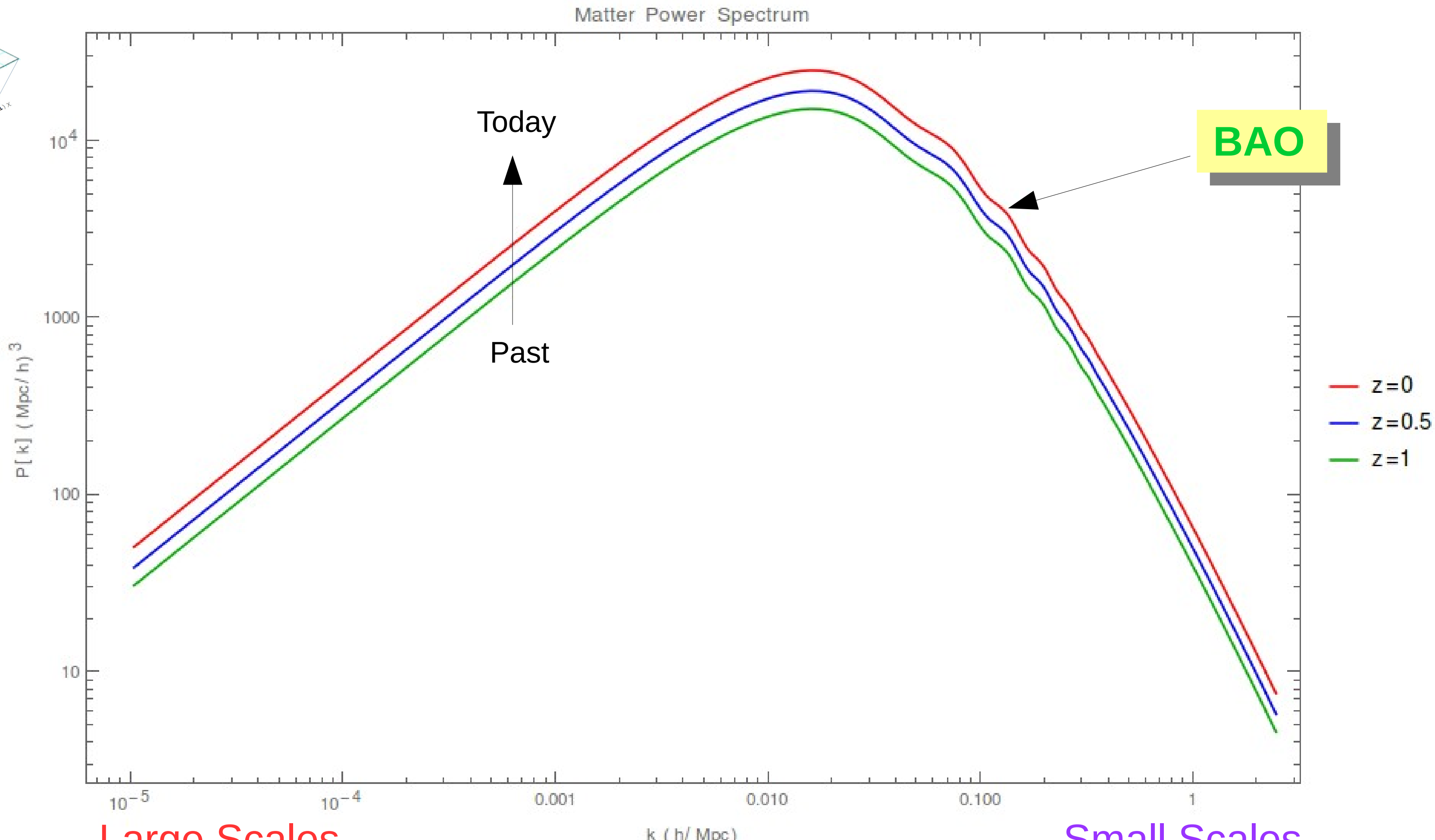
Hubble's law

Hubble flow (recession) **velocity:** $\frac{\partial d(t)}{\partial t} \equiv \upsilon = H\,d(t)$

Peculiar velocity (anisotropy): $\upsilon_{pec} = a(t)\frac{\partial \chi}{\partial t} \rightarrow \upsilon = H\,d(t) + \upsilon_{pec}$



The Power Spectrum



Large Scales

Small Scales

Planck 2015 cosmology

CAMB [A. Lewis]

Description of observations

General Relativity
How to measure distances?

Flat, homogeneous and isotropic universe $$ds^2 = -e^{2\Psi} dt^2 + a^2 e^{2\Phi} dx^2$$

Variational principle

Fluid equations + **Poisson constraint**
energy density ρ and pressure P

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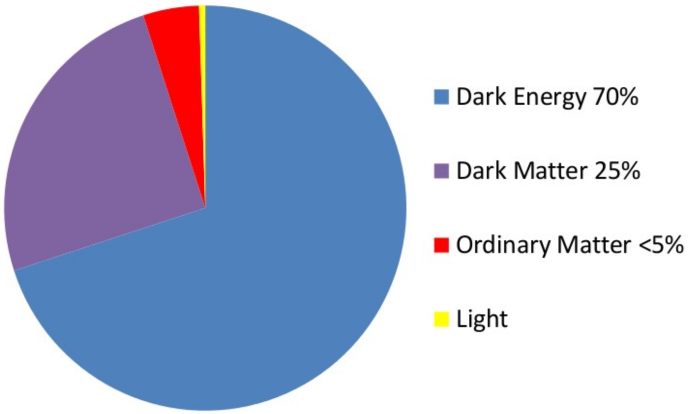

Description of observations

General Relativity
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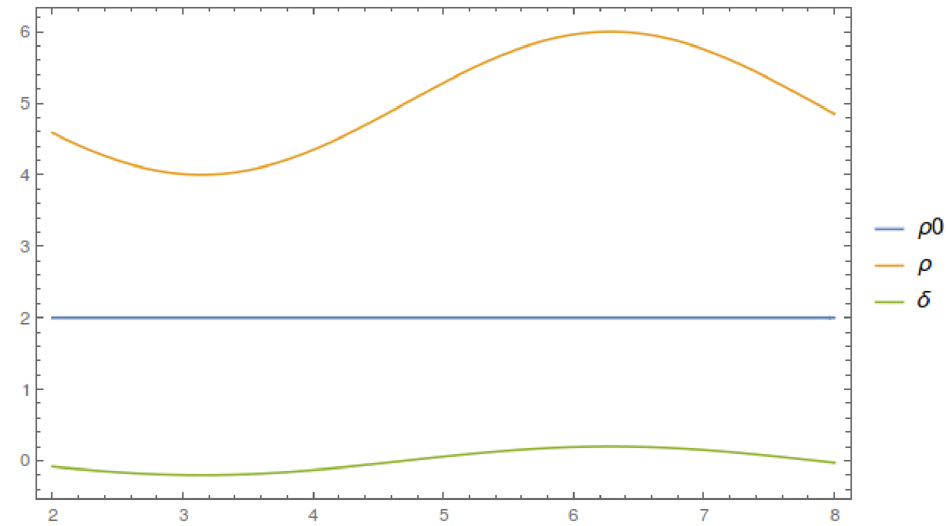
Variational principle

Fluid equations + Poisson constraint
energy density ρ and pressure P

Dark Matter (non relativistic)

$$P = 0$$
$$\rho = \rho_0 + \delta\rho$$
$$\delta = \frac{\delta\rho}{\rho}$$

Density contrast



Fluid equations

Density conservation

$$\dot{\delta} + \underline{\nabla} \cdot \left((1+\delta) \underline{v} \right) = 0 \qquad [1]$$

Newton's law

$$\dot{\underline{v}} + (\underline{v} \cdot \underline{\nabla}) \underline{v} + 2 H \underline{v} - \frac{1}{a^2} \underline{\nabla} \Phi = 0 \qquad [2]$$

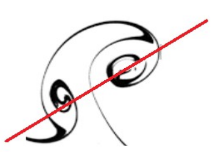
Poisson's equation

$$\frac{1}{a^2} \nabla^2 \Phi = -\frac{3 H^2}{2} \Omega_m \delta \qquad [3]$$

Cooking up:

$$\Omega_m = \rho_m / 3 H^2 M_p^2$$

1. Re-arrange terms to get a single second-order equation for delta.
2. Fourier space.
3. Time derivatives $\dot{\square} \equiv \frac{\partial}{\partial t}$ → redshift derivatives $\square' \equiv \frac{\partial}{\partial z} = -\frac{H}{a} \frac{\partial}{\partial t}$
4. Assumptions Perfect fluid behaviour, non-relativistic limit and negligible vorticity.



The matter density equation

$$\epsilon = -\frac{\dot{H}}{H^2}$$

$$\delta_k'' - \frac{1-\epsilon}{1+z}\delta_k' - \frac{3}{2}\frac{\Omega_m}{(1+z)^2}\delta_k = -\int \frac{d^3\underline{q}\, d^3\underline{s}}{(2\pi)^6}(2\pi)^3\delta(\underline{k}-\underline{q}-\underline{s})S_2(\underline{q},\underline{s}) - \int \frac{d^3\underline{q}\, d^3\underline{s}}{(2\pi)^6}(2\pi)^3\delta(\underline{k}-\underline{q}-\underline{s})\int \frac{d^3\underline{t}\, d^3\underline{\sigma}}{(2\pi)^6}(2\pi)^3\delta(\underline{s}-\underline{t}-\underline{\sigma})S_3(\underline{q},\underline{s},\underline{t},\underline{\sigma}) \quad [4]$$

Linear equation

Non-linear contribution

Eulerian Perturbation

Solving the equation...via expansion in delta

$$\delta = \delta^{(1)} + \delta^{(2)} + \delta^{(3)}$$

Linear solution

$$\delta^{(1)}(k,z) = D(z)\tilde{\delta}_k$$

$$\tilde{\delta}_k = \delta_k(\tilde{z}=50)$$

$$D'' - \frac{1-\epsilon}{1+z}D' - \frac{3}{2}\frac{\Omega_m}{(1+z)^2}D = 0$$

Growth function

Second-order

Third-order solution

Insert	in	To get
$\delta_k^{(1)}$	$S_2(\underline{q},\underline{s})$	$\delta^{(2)}(k,z)$
$\delta_k^{(1)},\delta_k^{(2)}$	$S_3(\underline{q},\underline{s},\underline{t},\underline{\sigma})$	$\delta^{(3)}(k,z)$

2-Point Correlation Function

$$\langle \delta^{(n)}(k_1, z)\,\delta^{(m)}(k_2, z)\rangle = (2\pi)^3\,\delta(\underline{k_1}+\underline{k_2})\,P_{nm}(k, z)$$

$$P^{SPT}_{1-loop}(k, z) = P_{11}(k, z) + P_{13}(k, z) + P_{22}(k, z) \qquad [5]$$

Tree level

$$P_{11}(k, z) = D(z)^2 \tilde{P}(k) \qquad [6]$$

1 loop corrections

$$P_{22}(k, z) = D_A^2(z) P_{AA}(k, z) + D_A(z) D_B(z) P_{AB}(k, z) + D_B^2(z) P_{BB}(k, z) \qquad [7]$$

$$\begin{aligned} P_{13}(k, z) = D(z)\tilde{P}(k)[(D_D(z) - D_J(z)) P_D(k, z) + D_E(z) P_E(k, z)] \\ + D(z)\tilde{P}(k)[(D_F(z) + D_J(z)) P_F(k, z) + D_G(z) P_G(k, z)] \\ + D(z)\tilde{P}(k)[\frac{D_J(z)}{2}(P_{J2}(k, z) - 2P_{J1}(k, z))] \quad . \end{aligned} \qquad [8]$$

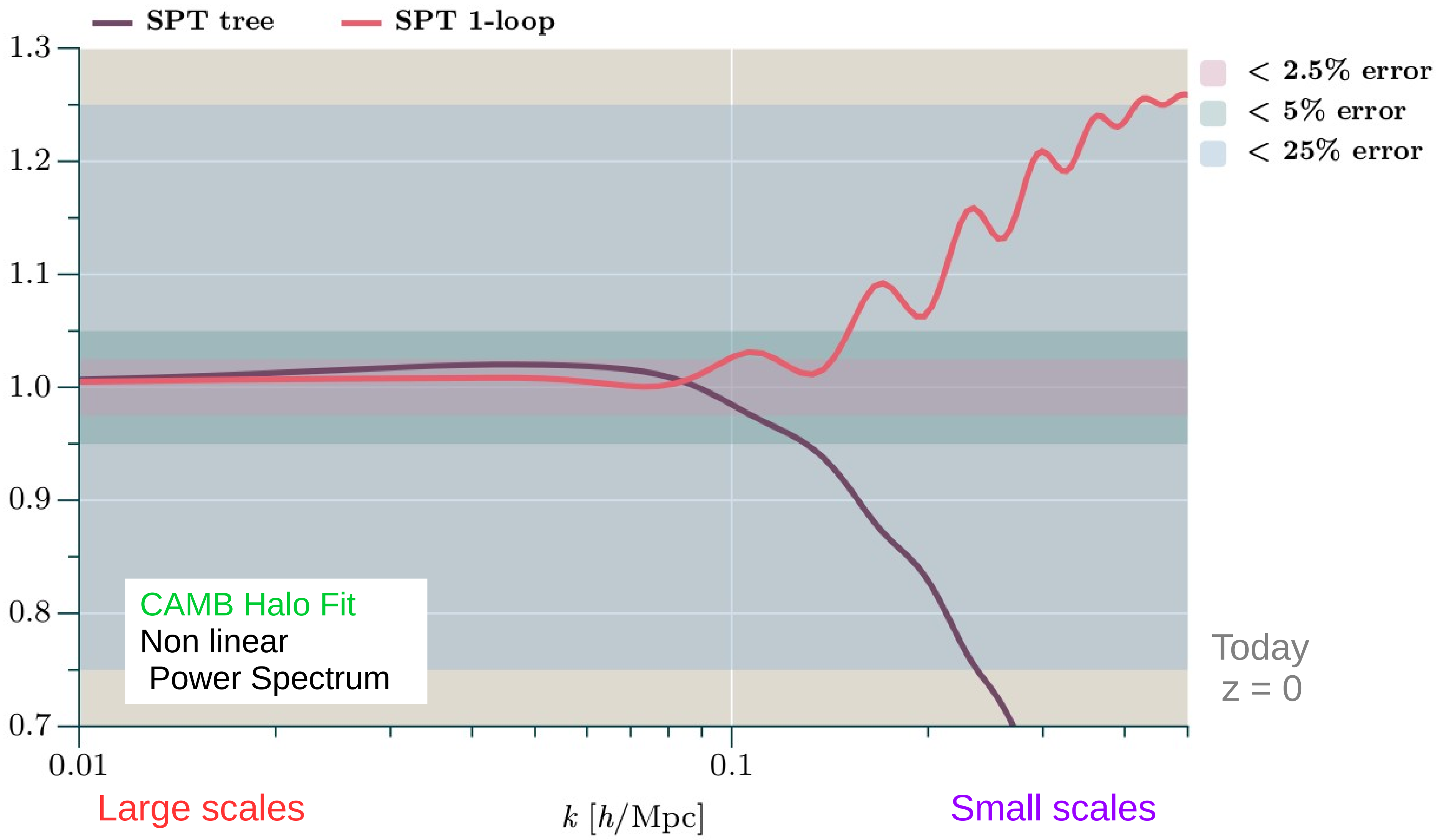
$D_{IJ} \wedge D_I$ *growth functions*

$P_{IJ} \wedge P_I$ *loop integrals*

$\tilde{P}(k) \equiv P(k, \tilde{z} = 50)$

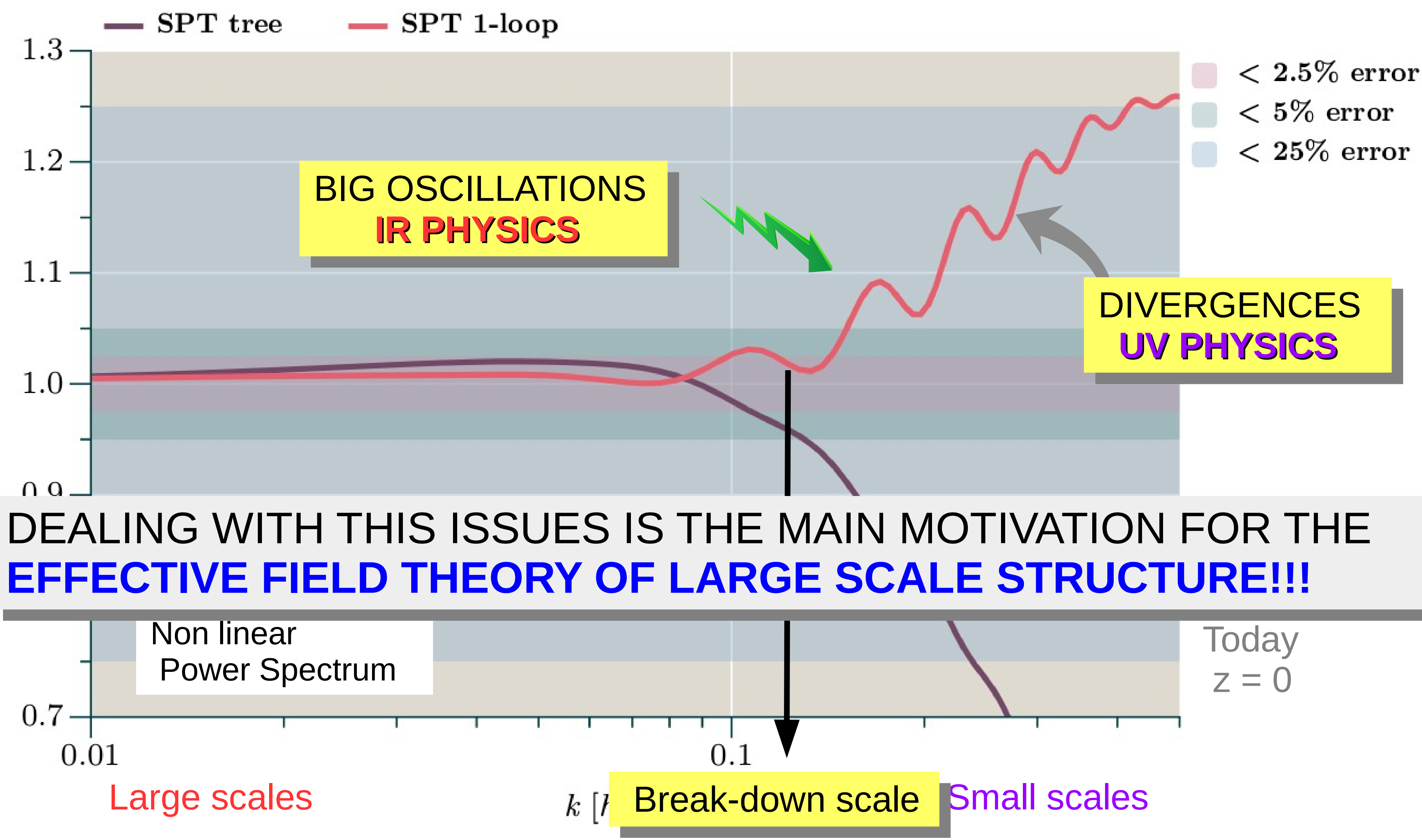
Initial power spectrum by CAMB

1-loop matter power spectrum



Motivation...

1-loop matter power spectrum



DEALING WITH THIS ISSUES IS THE MAIN MOTIVATION FOR THE **EFFECTIVE FIELD THEORY OF LARGE SCALE STRUCTURE!!!**

- We present the Standard Theory of Cosmological Perturbations.
- We derive the fluid equations for a Dark Matter component.
- We solve the 1 loop equation of the Matter Density Contrast and compute the 2-point correlation function.
- We extract the 1 loop matter power spectrum and compare with the non-linear power spectrum from CAMB.
- **Issues**:
  - Standard Perturbation Theory breaks down at scales of interest (new and future surveys).
  - Effects from UV physics make the power spectrum diverge, so results become untrustworthy.
  - IR effects over-amplify the effect of Baryon Acoustic Oscillations.

PART II...

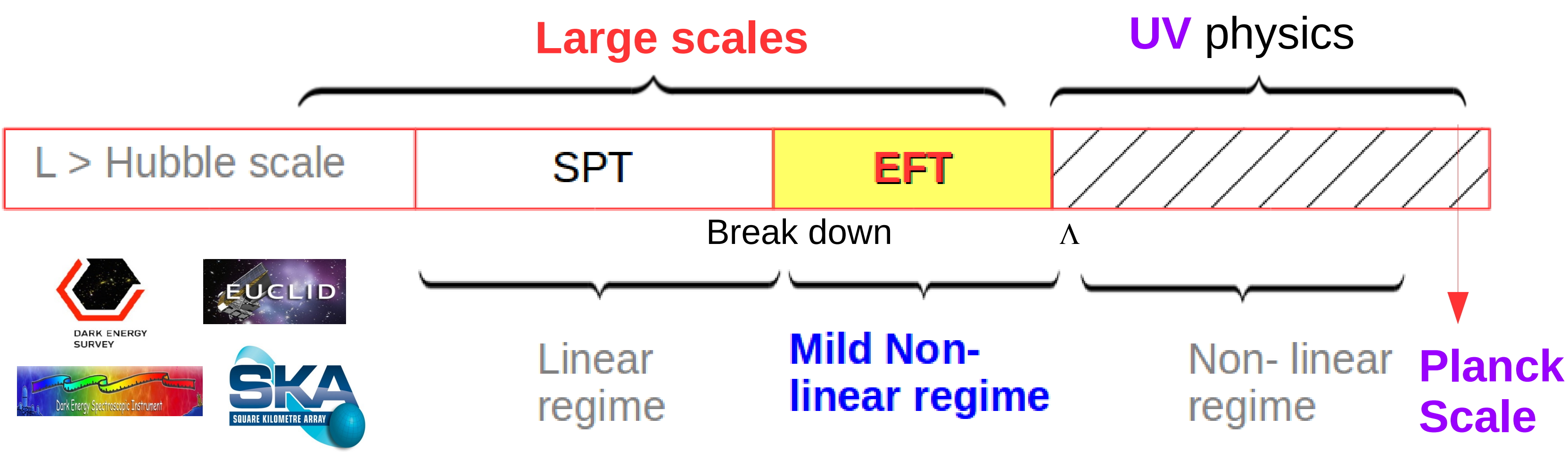
Effective Field Theory of Large Scale Structure in Real Space

EFFECTIVE FIELD THEORY OF LARGE SCALE STRUCTURE IN REAL SPACE

1. Effective Field Theory Formalism
2. Counter-term concept
3. The matter density equation
4. Dealing with UV physics
5. Fit the counter-term
6. Dealing with IR physics
7. 1-loop mater power spectrum

MOTIVATION FOR EXTENDING THE THEORY TO REDSHIFT SPACE

Effective Field Theory formalism



Parametrize our **UV ignorance**

- Ultimate high-energy theory of Gravity (Planck scale).
- Halo and galaxy formation, gas dynamics, active galactic nuclei feedback...

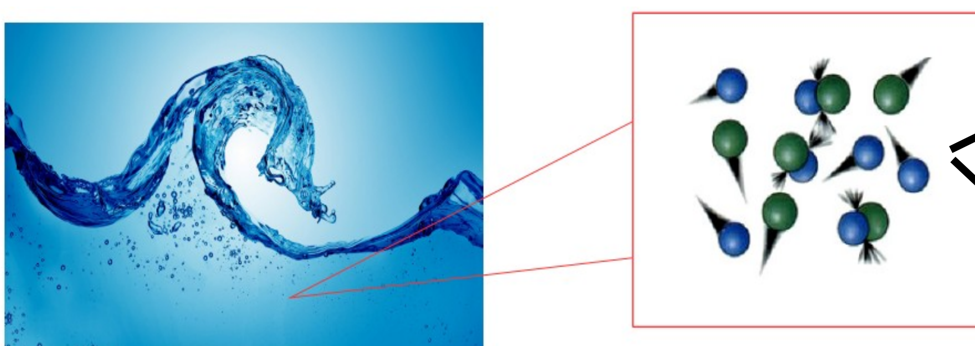
→ **Counter-term**

Counter-term concept

1 **Re-normalization of bare operators** (Analogue of Particle Physics)

$$\delta^R(k,z) = \delta_\Lambda(k,z) - \frac{c_\delta^2(z)}{k_{NL}^2} k^2 \delta_\Lambda(k,z)$$

2 **NO perfect fluid approximation** (Effective energy momentum tensor)



SMALL-SCALE INTERACTIONS

Viscosity
(1-loop)

Diffusion/dissipation
(not at 1-loop)

Dealing with UV physics

$$\delta_k'' - \frac{1-\epsilon}{1+z}\delta_k' - \frac{3}{2}\frac{\Omega_m}{(1+z)^2}\delta_k =$$

Linear equation

$$\frac{Z_\delta}{H^2}k^2\delta_k + \frac{Z_v}{H}k^2\delta_k'$$

Counter-term

$$-\int\frac{d^3\underline{q}\,d^3\underline{s}}{(2\pi)^6}(2\pi)^3\delta(\underline{k}-\underline{q}-\underline{s})S_2(\underline{q},\underline{s})$$
$$-\int\frac{d^3\underline{q}\,d^3\underline{s}}{(2\pi)^6}(2\pi)^3\delta(\underline{k}-\underline{q}-\underline{s})\int\frac{d^3\underline{\tau}\,d^3\underline{\sigma}}{(2\pi)^6}(2\pi)^3\delta(\underline{s}-\underline{\tau}-\underline{\sigma})S_3(\underline{q},\underline{s},\underline{\tau},\underline{\sigma})$$ [9]

Non-linear contribution

1. **Solve the equation** $\delta_{EFT} = \delta^{(1)} + \delta^{(2)} + \delta^{(3)} + \delta_{CT}$
2. **2-point correlation functions** $\langle\delta^{(n)}(k_1,z)\delta^{(m)}(k_2,z)\rangle = (2\pi)^3\delta(\underline{k_1}+\underline{k_2})P_{nm}(k,z)$
3. **The 1-loop matter power spectrum**

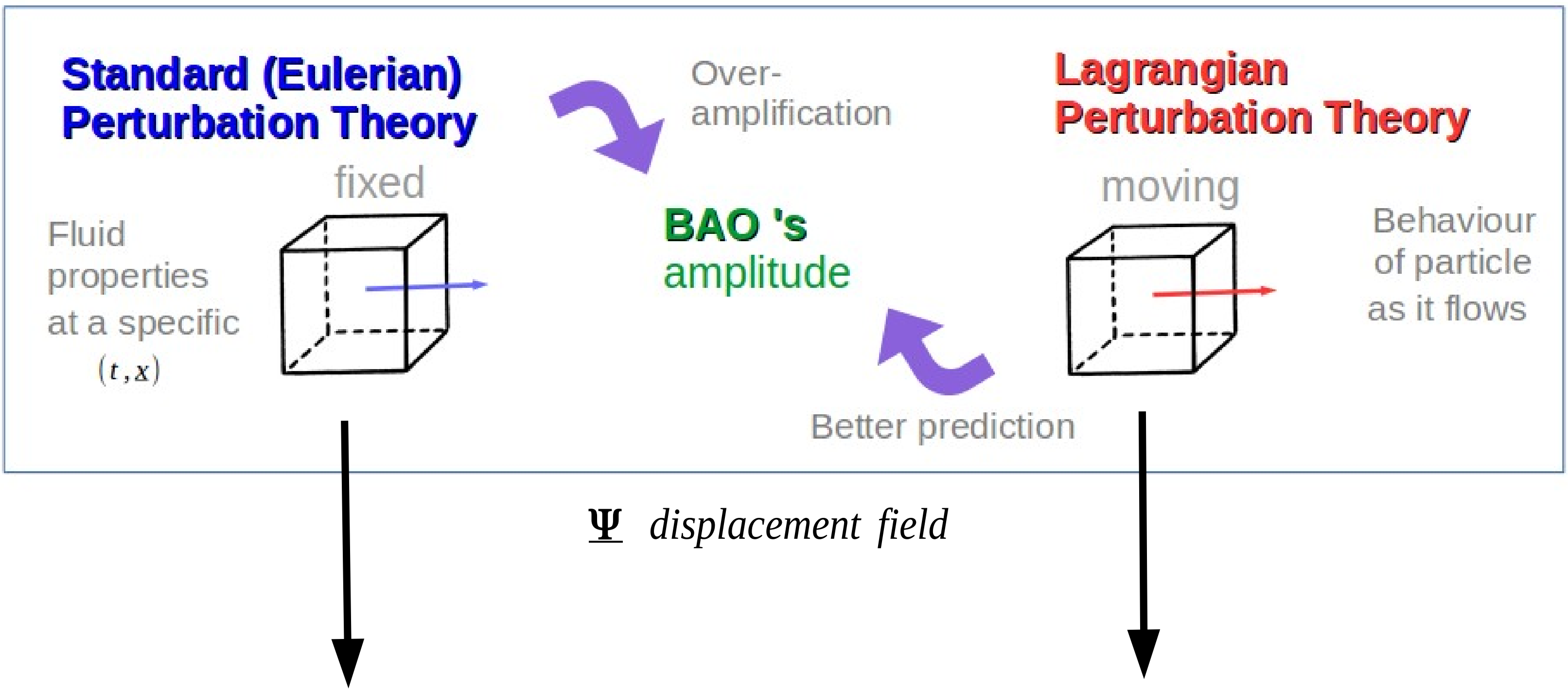
$$P^{EFT}_{1-loop}(k,z) = P^{SPT}_{1-loop}(k,z) - 4\pi D(z)^2\frac{c_s^2(z)}{k_{NL}^2}k^2\tilde{P}(k)$$ [10]

Re-normalisation factor **(CAMB)**

$$\frac{c_{2,\delta}}{k_{NL}^2} = 1.57\,Mpc^2/h^2$$

Dealing with IR physics

$$\underline{r}(x_q,t) = x_q + \Psi(x_q,t)$$



$\underline{\Psi}$ *displacement field*

Density and **displacement** fields treated perturbatively

Only **density** fields treated perturbatively

Dealing with IR physics

IR re-summation

[Vlah et al.]

Power Spectrum

P_{1-loop}

$\widetilde{P} \rightarrow \widetilde{P}_{NW}$

$P_{11},\ P_{22},\ P_{13}\ldots$

$\Delta P_w(k,z) \equiv P(k,z) - P_{NW}(k,z)$

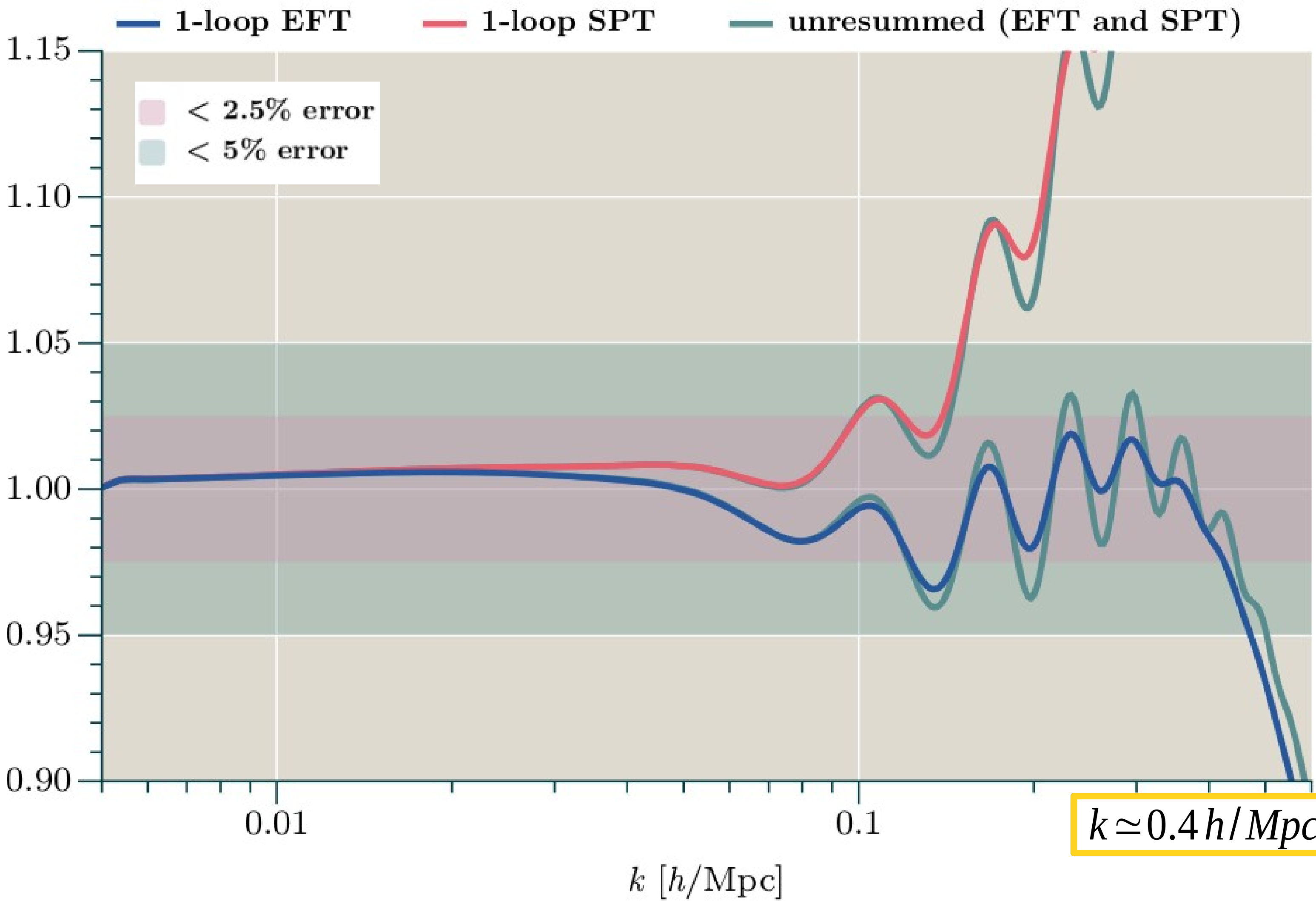
$$P^{IR}(k) = \int d^3 \underline{x}_q \, e^{-i\underline{k}\cdot\underline{x}_q} \mathcal{K}(\underline{k}, \underline{q}, \underline{\Psi}(\underline{q})) \rightarrow P^{IR}(k) = P_{NW}(k) + \int d^3 \underline{x}_q \, e^{-i\underline{k}\cdot\underline{x}_q} \mathcal{K}_W(\underline{k}, \underline{q}, \underline{\Psi}(\underline{q}))$$

Lagrangian Perturbation

$$P^{IR}(k,z) = P_{NW}(k,z) + e^{\Sigma^2 k^2}\left(\Delta P_{1-loop,\,NW}(k,z) + \Sigma^2 k^2 \Delta P_{11,\,w}(k,z)\right)$$ [11]

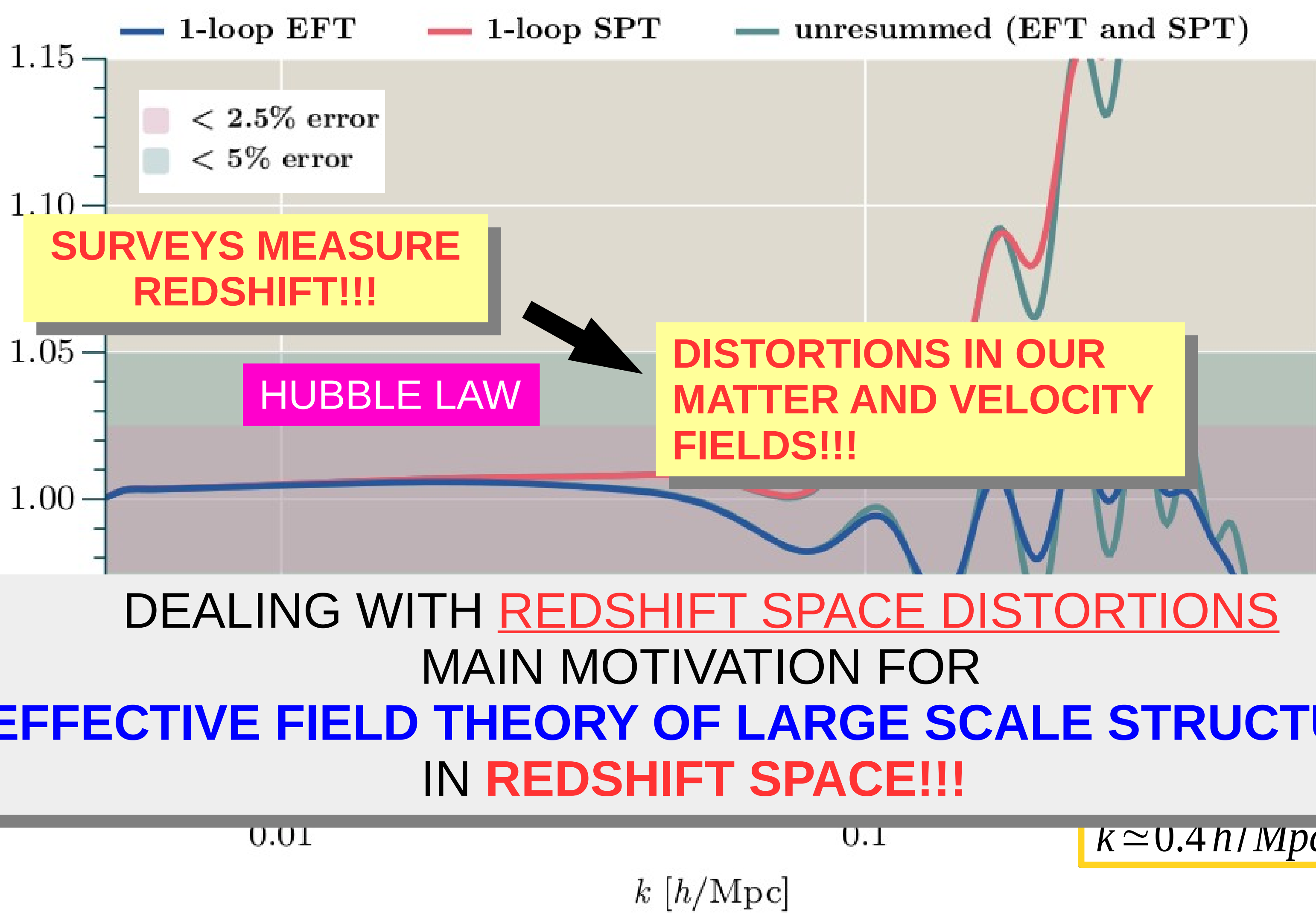
OSCILLATIONS ARE DAMPED

1-loop matter power spectrum



$k \simeq 0.4\, h/Mpc$

1-loop matter power spectrum



SURVEYS MEASURE REDSHIFT!!!

HUBBLE LAW

DISTORTIONS IN OUR MATTER AND VELOCITY FIELDS!!!

$k \simeq 0.4\, h/Mpc$

DEALING WITH REDSHIFT SPACE DISTORTIONS
MAIN MOTIVATION FOR
EFFECTIVE FIELD THEORY OF LARGE SCALE STRUCTURE
IN **REDSHIFT SPACE!!!**

- We use the Effective Field Theory framework to deal with the issues encountered by the Standard Perturbation Theory:
  - Breaks down at scales of interest (new and future surveys).
  - Effects from UV physics make the power spectrum diverge, so results become untrustworthy.
- We solve the 1 loop equation of the Matter Density Contrast and compute the 2-point correlation function.
- We extract the 1 loop matter power spectrum and compare with the non-linear power spectrum from CAMB.
- We fit the counter-term.
- We apply a re-summation scheme to account correctly for the Baryon Acoustic Oscillations.
- **Issues**:
  - Real surveys provide observational data in redshift space.
  - Redshift Space Distortions need to be addressed.

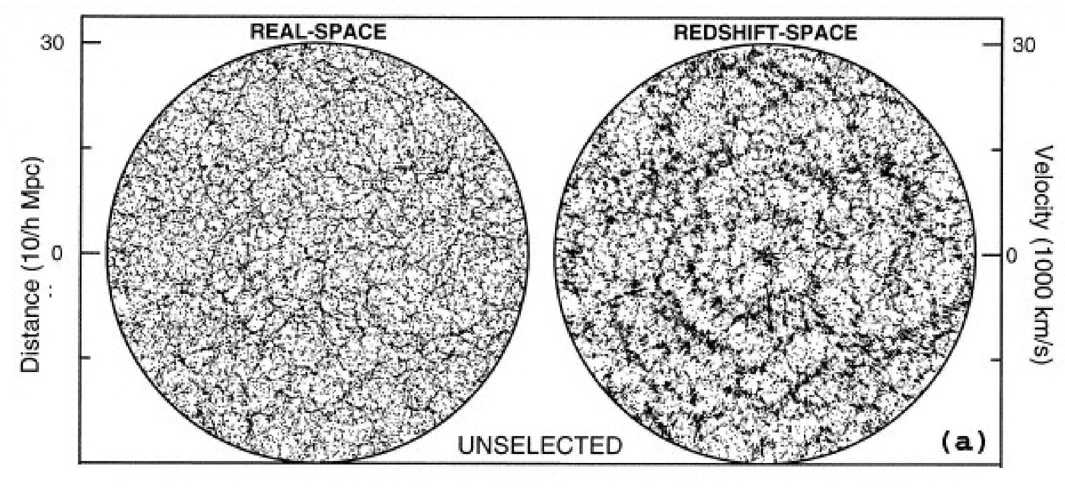
PART III...

Effective Field Theory of Large Scale Structure in Redshift Space

EFFECTIVE FIELD THEORY OF LARGE SCALE STRUCTURE IN REDSHIFT SPACE

1. Redshift Space Distortions
2. Density contrast in Redshift Space
3. 1-loop mater power spectrum in Redshift Space
4. Fit Counter-terms
5. Results and comparison

Redshift Space Distortions



Real space ⟷ **Redshift space**

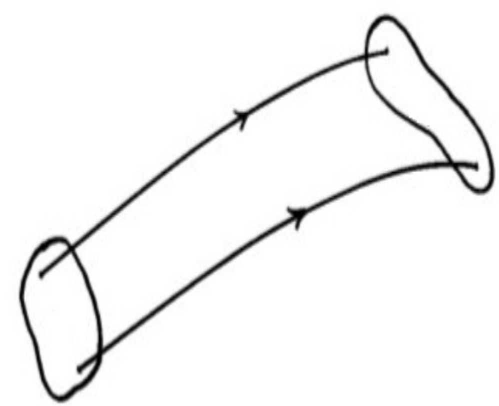
$\{r^i\}$ $\{s^i\}$

$$\underline{s} = \underline{r} + \frac{\hat{\underline{r}} \cdot \underline{\upsilon}}{H} \hat{\underline{r}}$$

Redshift is affected by **random motions** along line of sight

$$\mu = \hat{k} \cdot \hat{r}$$

CONSERVATION OF DENSITY

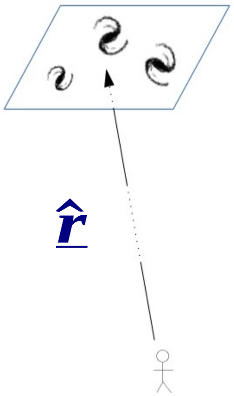


$$\rho(\underline{s}) d^3\underline{s} = \rho(\underline{x}) d^3\underline{x}$$

[12]

Density contrast in Redshift Space

$$\delta_s(\underline{k}) = \int d^3\underline{x}\, e^{-i\underline{k}\cdot\underline{x}}[e^{-i\underline{k}\cdot\underline{x}(\underline{x}\cdot\underline{\upsilon})} - 1][1+\delta(\underline{x})] \qquad [13]$$



Fourier space

$$\begin{aligned}[\delta_s]_{\underline{k}} = [\delta]_{\underline{k}} &- \frac{i}{H}(\underline{k}\cdot\hat{\underline{r}})[\hat{\underline{r}}\cdot\underline{\upsilon}]_{\underline{k}} - \frac{i}{H}(\underline{k}\cdot\hat{\underline{r}})[\hat{\underline{r}}\cdot\underline{\upsilon}\delta]_{\underline{k}} \\ &+ \frac{1}{2!\,H^2}(\underline{k}\cdot\hat{\underline{r}})^2[(\hat{\underline{r}}\cdot\underline{\upsilon})^2]_{\underline{k}} + \frac{1}{2!\,H^2}(\underline{k}\cdot\hat{\underline{r}})^2[(\hat{\underline{r}}\cdot\underline{\upsilon})^2\delta]_{\underline{k}} \\ &+ \frac{i}{3!\,H^3}(\underline{k}\cdot\hat{\underline{r}})^3[(\hat{\underline{r}}\cdot\underline{\upsilon})^3]_{\underline{k}} + O(4) \quad .\end{aligned} \qquad [14]$$

$[f]_k \equiv$ *Fourier transform*

1-loop matter power spectrum in Redshift Space

1. Compute the 2-point correlation function $\langle \delta_s(\underline{k_1}) \delta_s(\underline{k_2}) \rangle$

2. Get all contributions to the power spectrum $P_{s,11}(k,\mu,z)$, $P_{s,22}(k,\mu,z)$, $P_{s,13}(k,\mu,z)$...

Legendre decomposition

(Surveys)

$$P_{s,l}(k,z) = \frac{2l+1}{2} \sum_{n=0}^{3} \int_{-1}^{1} \mu^{2n} \mathscr{L}_l(\mu) P_{2n,s}(k,z)$$

$\mathscr{L}_l(\mu)$ *Legendre polynomials*

l=0, 2 and ***4*** modes

3. Fit counter-terms using N-body simulations (UV divergences)

$$P_{s,l}(k,z) = P_{s,l}^{SPT} - 2D(z)^2 \frac{d_{\delta_s,l}}{k_{NL}^2} k^2 \tilde{P}_l(k)$$

Three counter-terms
(N-BODY SIMULATIONS)

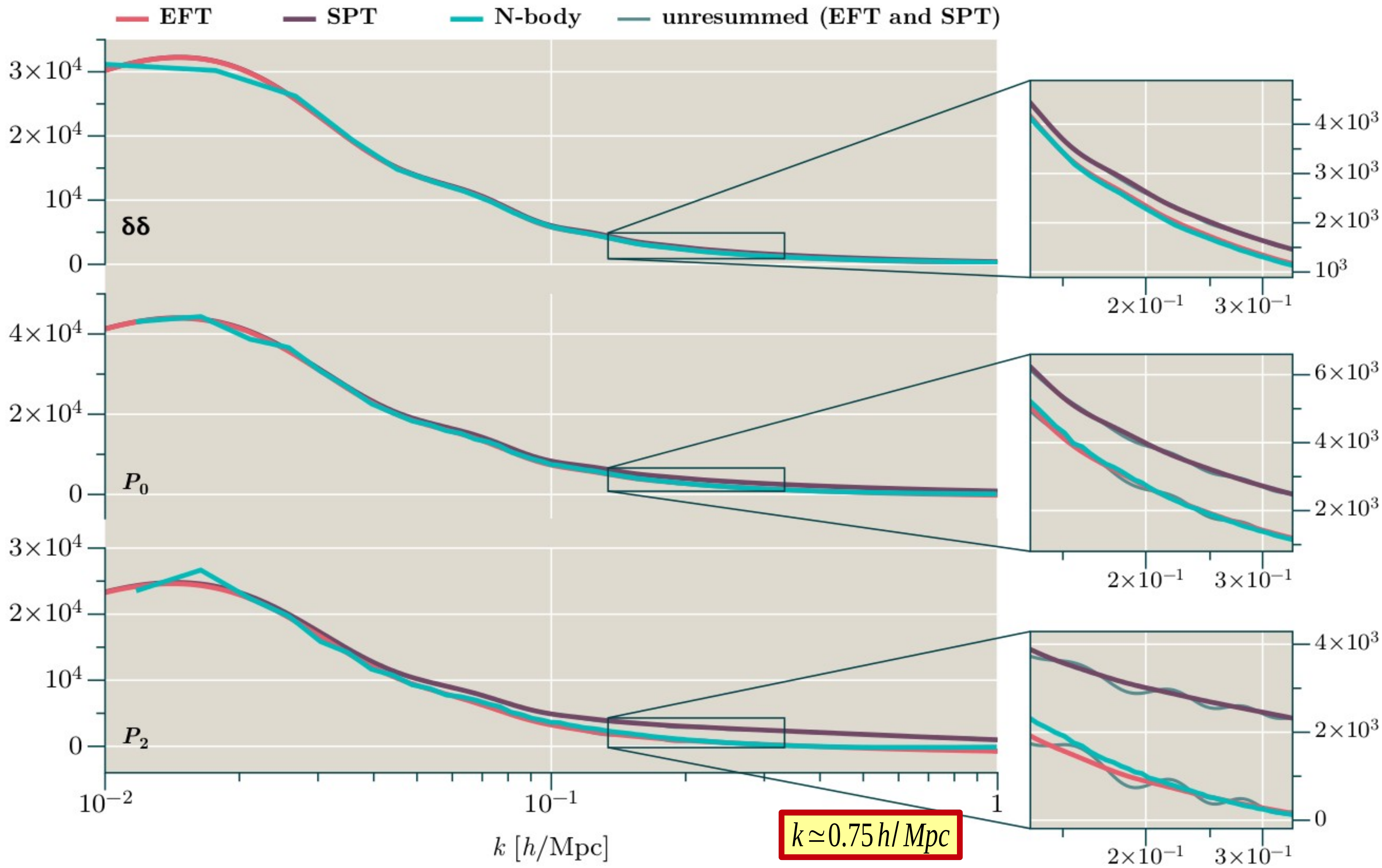
$$\frac{d_{\delta,0}}{k_{NL}^2} = 1.88\, Mpc^2/h^2$$

$$\frac{d_{\delta,2}}{k_{NL}^2} = 15.8\, Mpc^2/h^2$$

$$\frac{d_{\delta,4}}{k_{NL}^2} = 6.43\, Mpc^2/h^2$$

4. Apply IR re-summation scheme.

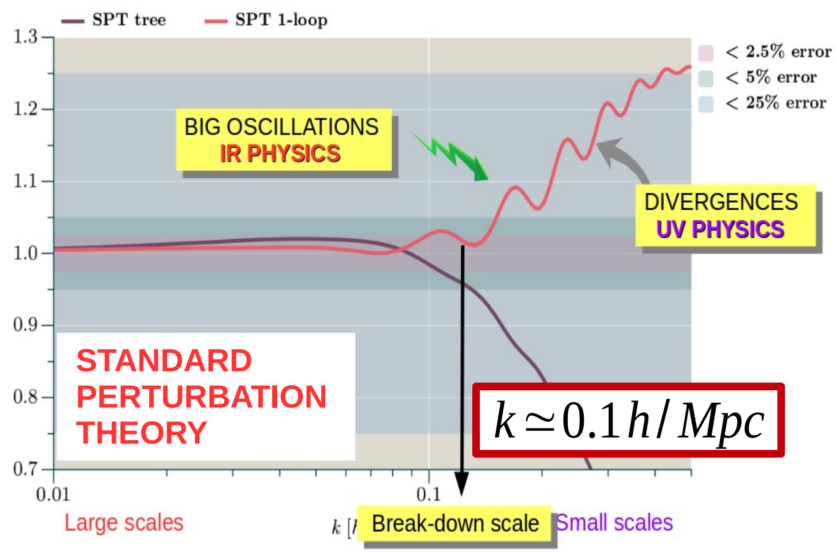
Results and comparison



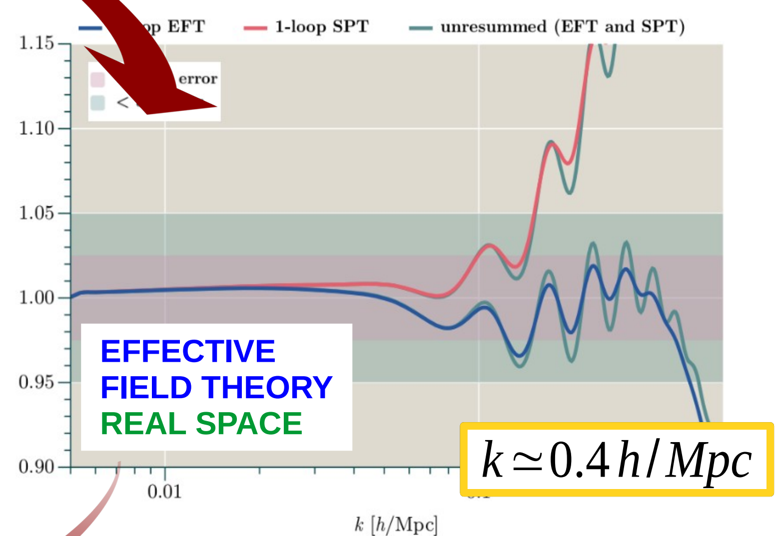
- Effective Field Theory in Real Space to deal with the issues encountered by the Standard Perturbation Theory: predictions are reliable for larger values of *k* since counter-terms absorb divergences from high-energy Physics.
- Real surveys provide observational data in redshift space so we need to translate every result into Redshift Space.
- Power Spectra given by surveys appear decomposed in their different Legendre multipoles.
- We decompose the redshift-space 1-loop matter power spectrum in its monopole, cuadrupole and hexadecapole components.
- We fit one counter-term for each multipole using N-body simulations.
- We apply the IR re-summation scheme.
- We compare our results with N-body simulations and see the success of the re-summed predictions of EFToLSS in Redshift-Space and validity up to $k \simeq 0.75\, h/Mpc$.

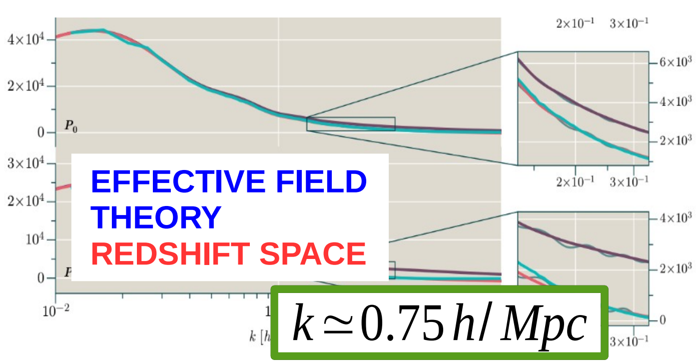
PART IV...

Recap, conclusions and prospects



Counter-term
IR re-summation





Redshift-space distortions

Conclusions

- The Effective Field Theory framework produces fits that extend the reach of Standard Perturbation Theory by a factor of a few in k.
- However, the practical value of these fitting functions has not yet been demonstrated.
- Without a prediction for the time-dependence of the counter-terms, we are obliged to fit independently at each redshift.
- The values of the counter-terms vary even between relatively nearby cosmologies.
- Some ideas to compute covariance matrices that extend to small scales but accurately modelling redshift-space measurements is extremely expensive (much more than for the case in real space).

Prospects

- **Precission test of Modified Gravity**: we could use the cosmological concordance model as a null test and introduce Modified Gravity Theories to analyse screening mechanism → merging Effective Field Theory of Dark Energy and of Large Scale Structures [L. Fonseca de la Bella and L.Perenon].

Dealing with UV physics

$$loop\ \ integrals \supseteq \int^{\Lambda} \frac{d^3 q}{(2\pi)^3} \tilde{P}(q) \mathcal{K}(\underline{k}, \underline{q}; z)$$

$$= \underbrace{\int_0^{\tilde{k}} \frac{d^3 q}{(2\pi)^3} \tilde{P}(q) \mathcal{K}(\underline{k}, \underline{q}; z)}_{Linear\ \ regime} + \underbrace{\int_{\tilde{k}}^{\Lambda} \frac{d^3 q}{(2\pi)^3} \tilde{P}(q) \mathcal{K}(\underline{k}, \underline{q}; z)}_{Mild\ \ non-linear\ \ regime}$$

$$= f(z) Z(\Lambda) k^2 + b(z) \tilde{k}^3 + O(k^4)$$

Unreliable!

$$P_{CT}(k,z) = -\,2 D(z)^2 \frac{c_{2,\delta}(z)}{k_{NL}^2} k^2 \tilde{P}(k)$$

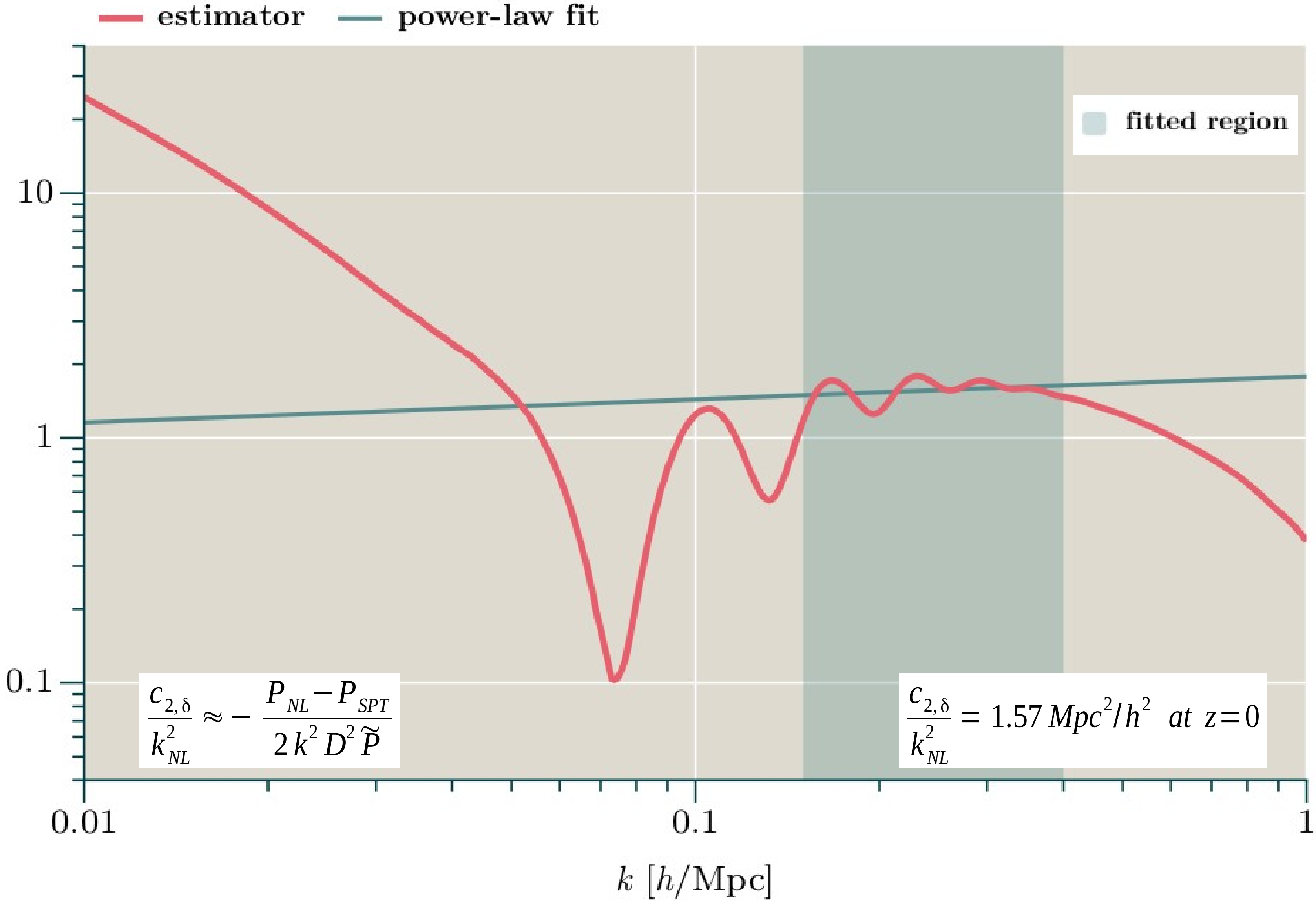
$$P_{13}(k,z) \underbrace{\supseteq}_{UV} 2 D(z)^2 F(z) Z_\delta(\Lambda) k^2 \tilde{P}(k)$$

UV limit → Taylor *k/q << 1*

COUNTER-TERMS absorb the divergences
must share same time dependence up to a constant

Re-normalisation factor
fixed CAMB HALOFIT
(data, simulations)

Fit the counter-term



Fit Counter-terms