

The amazing world of
*EFFECTIVE FIELD THEORY of
LARGE SCALE STRUCTURES in
REDSHIFT SPACE*

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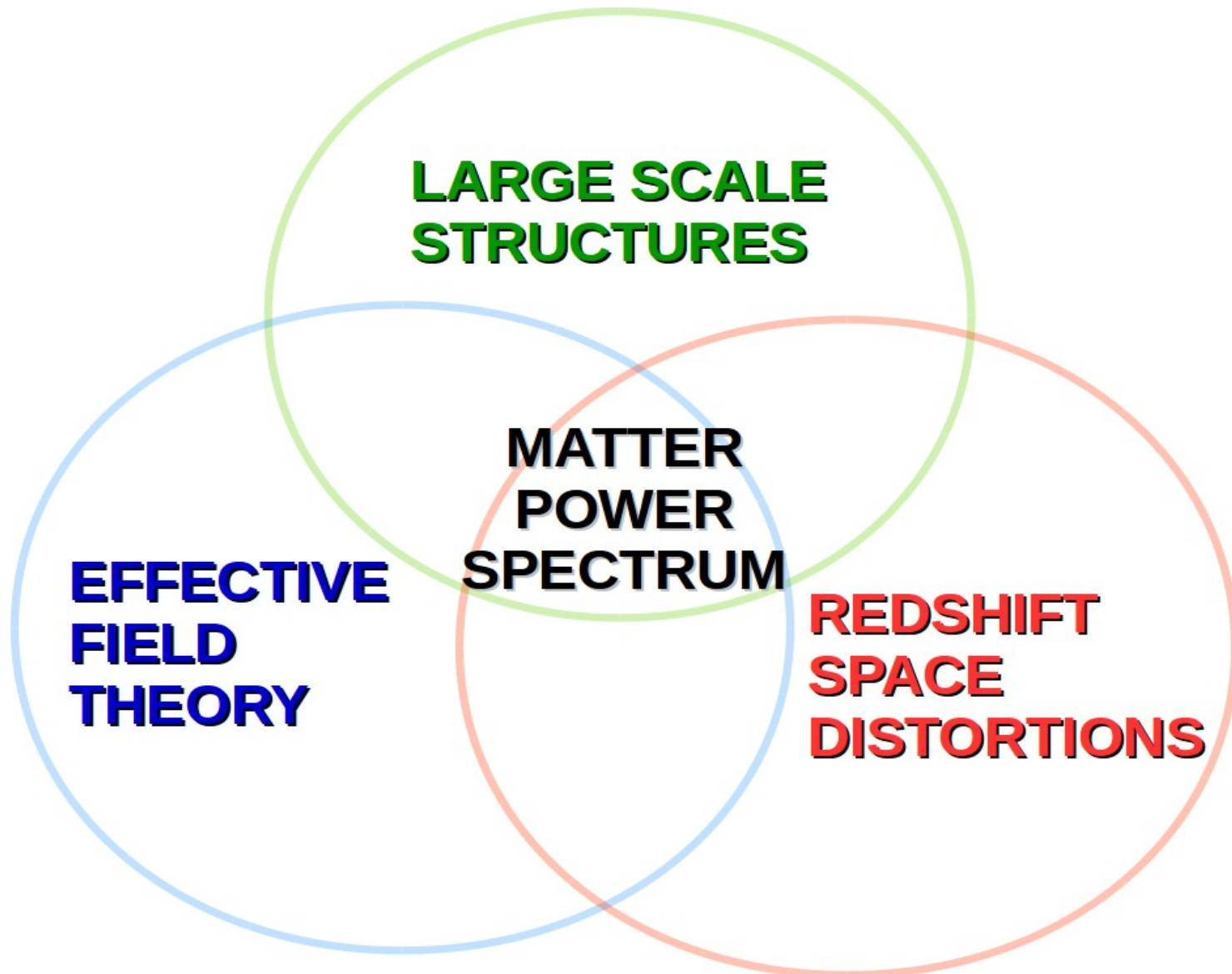
In collaboration with D.Seery and D. Regan



April, 22nd



...we'll talk about



I. Introduction-

- 1) Large Scale Structures
- 2) Effective Field Theory formalism
- 3) Redshift Space Distortions

Large scale structures

...why is this important?

- Learn about our **Universe**
- Large scales
 - Most information for Cosmology.
 - Galaxy point-like object.
 - Universe as a fluid.
- Structures:
 - Clusters, filaments, voids...



Millenium simulation, Springer et al 2005

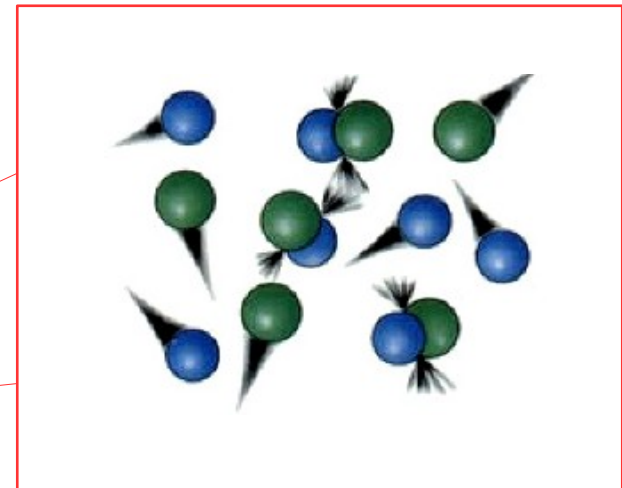
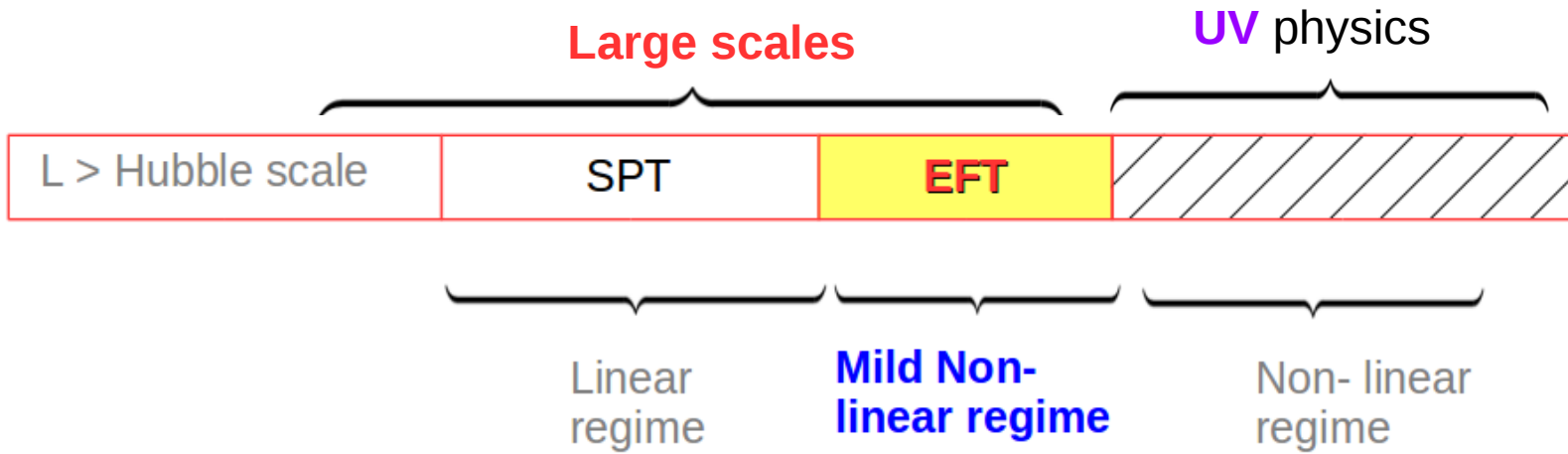
How?

- Simulations.
- Observational data!

Theoretical tool ?

Effective Field theory formalism

Carrasco et al 2012



Standard Perturbation

VS

Effective Field Theory

~~Standard Perturbation~~



EFToLSS

- **Good** in the linear regime (very large distances).
- Perfect fluid
- UV sensitive → **cut-off** dependent.

Bernardeau et al., 2002

- Probes scales of interest where **SPT breaks down**.
- Viscosity, dissipation...
- No UV sensitivity → **counterterms!**

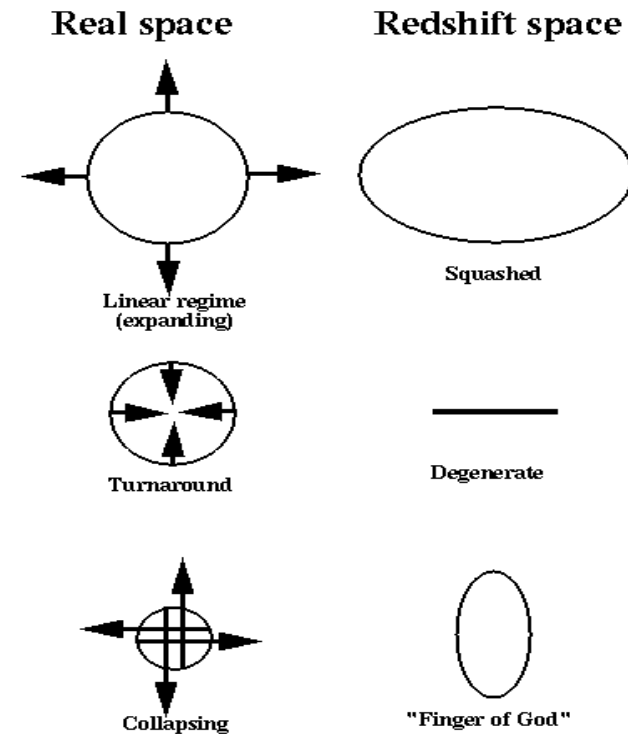
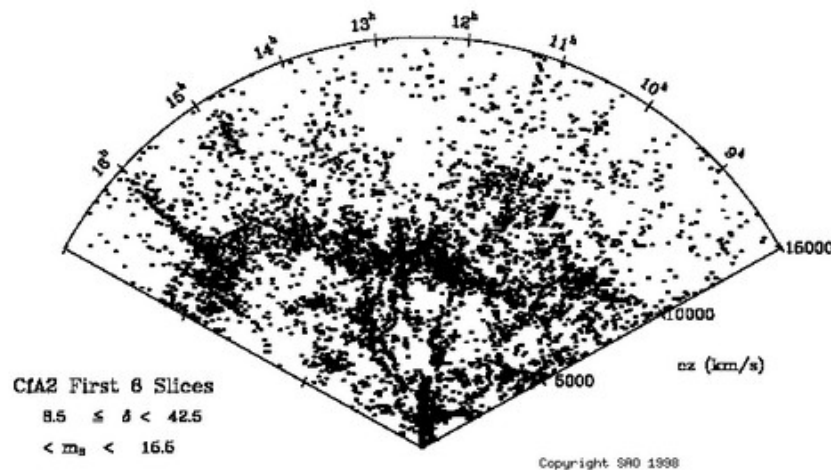
Carrasco et al 2012

Redshift space distortions

WHY?

- **Surveys** in redshift space.
- Redshift depends on **peculiar v** (along line of sight).
- Unknown **random motions** within clusters

Kaiser 1987



- Fingers of God.
- Learning about **velocity fields**.

II. EFToLSS in real space-

- 1) Fluid equations
- 2) Cosmological matter perturbations
- 3) Counterterms idea
- 4) Matter density contrast
- 5) 1-loop matter power spectrum
- 6) UV divergences

Fluid equations (General Relativity)

$$ds^2 = -e^{2\Psi(t,\vec{x})} dt^2 + a(t)^2 e^{2\Phi(t,\vec{x})} d\vec{x}^2$$

Equations of motion (expanding Universe)

Density
conservation

$$\partial_t \rho + \partial_m (\rho v^m) + 3\rho(H + \dot{\Phi}) = 0,$$

Newton's law

$$\partial_t v^i + v^m \partial_m v^i + 2H v^i + \frac{\partial^i \Psi}{a^2} = 0,$$

Poisson's equation

$$\frac{\partial^2 \Phi}{a^2} + \frac{\delta \rho}{2M_p^2} = 0.$$

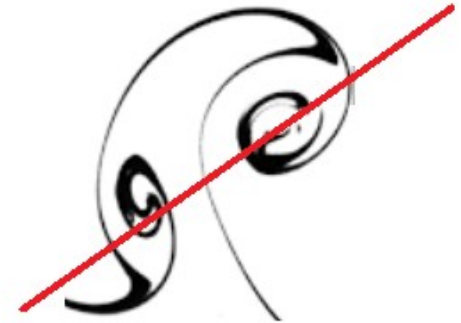
Cosmological matter perturbations

Density contrast

$$\delta = \delta\rho/\rho_0$$

velocity divergence

$$\Theta = \partial_i v^i$$



Standard Perturbation Theory

$$\dot{\delta} + \Theta = -\Theta\delta - (\partial_m \partial^{-2} \Theta) \partial^m \delta,$$

$$\dot{\Theta} + 2H\Theta + \frac{3}{2}H^2\Omega_M(z)\delta = -\partial_m \partial^{-2} \Theta \partial_m \Theta - \partial_i \partial_m \partial^{-2} \Theta \partial_m \partial_i \partial^{-2} \Theta.$$

WHAT'S NEXT? + COUNTERTERMS

Counterterms

(Effective energy momentum tensor)

Diffusion/dissipation
stochastic term

→ Random variable

Small scale interactions

Deviations from perfect fluid

Viscosity, speed of sound ...

→ Parametrise UV physics

$$Z_\delta \frac{\partial^2 \delta}{a^2}$$

$$Z_\Theta \frac{\partial^2 \Theta}{a^2}$$



Matter density contrast (Fourier space)

Linear equation

Counterterm

$$\epsilon = -\frac{\dot{H}}{H^2}$$

$$' \equiv d/dz$$

$$\delta_k'' - \frac{1-\epsilon}{1+z}\delta_k' - \frac{3}{2}\frac{\Omega_M(z)}{(1+z)^2}\delta_k = \frac{k^2}{H^2 a^2} \frac{Z_\delta \delta_k + Z_\Theta H(1+z)\delta_k'}{(1+z)^2}$$

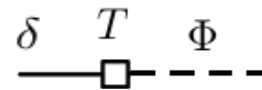
$$+ \int \frac{d^3 \vec{q} d^3 \vec{r}}{(2\pi)^6} (2\pi)^3 \delta(\vec{k} - \vec{q} - \vec{r}) \times [\alpha(\vec{q}, \vec{r}) \frac{3}{2} \frac{\Omega_M(z)}{(1+z)^2} \delta(\vec{q}) \delta(\vec{r}) + \gamma(\vec{q}, \vec{r}) \delta'(\vec{q}) \delta'(\vec{r})]$$

Non-linear contribution

Solving the equation...

Linear order

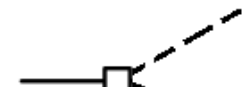
$$\delta^{(1)}(k, z) = g(z) \delta_k^*$$



$$\delta_k^* \equiv T_k(z_*) \Phi_k^{Prim}$$

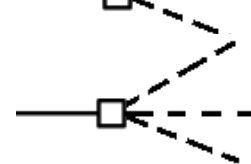
Quadratic order

$$\delta^{(2)}$$



Cubic order

$$\delta^{(3)}$$



Counterterm

$$\delta^{CT}$$



1 loop matter power spectrum

$$\langle \delta^{(n)}(k, z) \delta^{(n)}(k', z) \rangle = (2\pi)^3 \delta_D(\vec{k} + \vec{k}') P_{nn}(k, z)$$

The renormalised power spectrum at **1 loop**

$$\begin{aligned}
 P_{MM}(q) &= \text{---}\square\text{---}\square\text{---} + 2 \times \left(\text{---}\overset{\text{dashed}}{\square}\text{---}\square\text{---} + \text{---}\overset{\text{cross}}{\square}\text{---}\square\text{---} \right) + \text{---}\square\text{---}\overset{\text{dashed}}{\square}\text{---} \\
 &= \underbrace{P_{11}(q)}_{\text{Tree level}} + 2 \times \underbrace{(P_{13}(q) + P_{CT}(q))}_{\text{UV}} + \underbrace{P_{22}(q)}_{\text{UV}}
 \end{aligned}$$

Drop \rightarrow **2 loop + ...** and / or $P_{CT} \propto k^2$

\rightarrow $(k/k_{NL})^4 + \dots$ $P_{stoch} \propto k^4$

UV divergences

Example of loop integrals in momentum space found in P_{13}

$$\begin{aligned} I_{\alpha\alpha}(\Lambda) &= \int^{\Lambda} \frac{d^3\vec{q}}{(2\pi)^3} \mathcal{P}_R(\vec{q}) \alpha(\vec{k}, -\vec{q}) \alpha(\vec{k} - \vec{q}, \vec{q}) \\ &= \underbrace{\int_0^{k_*} \frac{d^3\vec{q}}{(2\pi)^3} \mathcal{P}_R(\vec{q}) \alpha(\vec{k}, -\vec{q}) \alpha(\vec{k} - \vec{q}, \vec{q})}_{\text{Linear regime, SPT, } \Lambda\text{-independent}} \\ &\quad + \underbrace{\int_{k_*}^{\Lambda} \frac{d^3\vec{q}}{(2\pi)^3} \mathcal{P}_R(\vec{q}) \alpha(\vec{k}, -\vec{q}) \alpha(\vec{k} - \vec{q}, \vec{q})}_{\text{Mild non-linear regime, UV sensitive}} \\ &= a_1(\Lambda) \cdot k^2 + b_1 \cdot k^3 + O(k^4). \end{aligned}$$

COUNTERTERMS 

Renormalisation

Drop \rightarrow 2 loop + ...
 $\rightarrow (k/k_{NL})^4 + \dots$

$$P_{\delta\delta|1\text{-loop}} = P_{11} + P_{13} + P_{CT}$$

P_{13} \rightarrow **SPT** linear regime \rightarrow **CUBIC** POLYNOMIAL IN k

\rightarrow **EFT** mild non-linear regime (Taylor expansion)

$$P_{13}(k, z) \approx P_{11}(k, z) k^2 h(z) \underbrace{\int^\Lambda \frac{d^3q}{2\pi^2} \mathcal{P}_{\mathcal{R}}(q)}_{\mathcal{A}(\Lambda)}$$

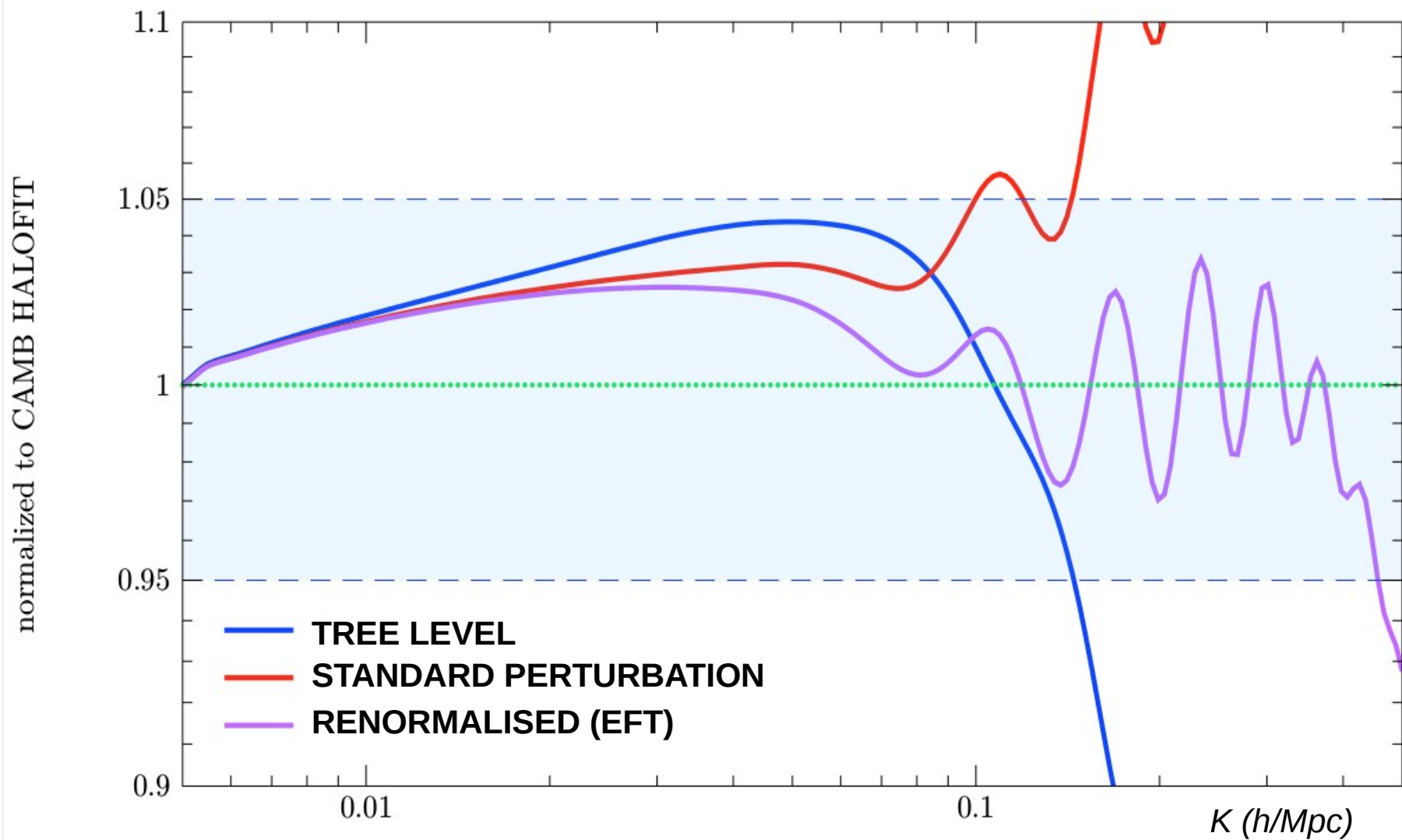
P_{CT} \rightarrow **UV LIMIT**
 It shares k & z dependence up to a constant

$$P_{\delta\delta|1\text{-loop}} = P_{11} [1 + \mathbf{c_s^2} h(z) k^2 + \mathbf{b} k^3 + O(k^4)]$$

\rightarrow **Renormalisation factor**
 fixed **CAMB HALOFIT** (data, simulations)

Results

1 LOOP MATTER POWER SPECTRUM

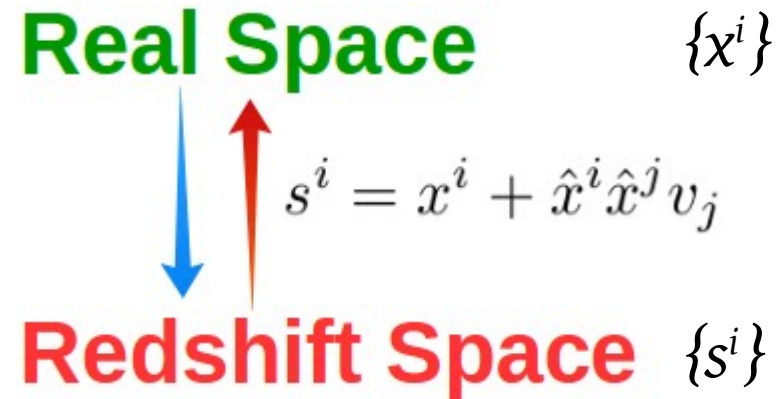
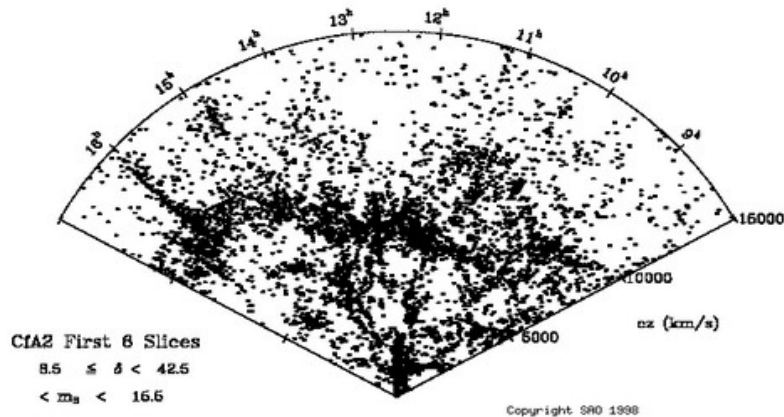


III. Redshift Space -

- 1) Redshift Space Distortions
- 2) Density contrast in redshift space

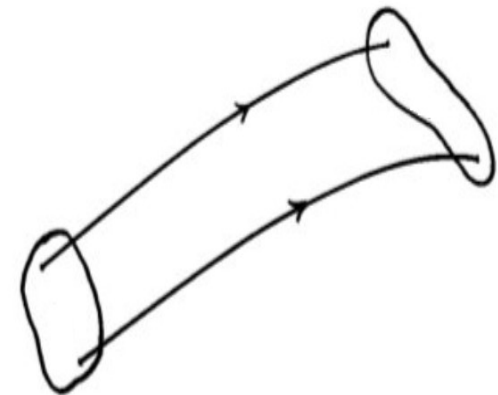
Redshift space distortions

Kaiser 1987



- Conservation of density

$$\rho(\vec{s})d^3\vec{s} = \rho(\vec{x})d^3\vec{x}$$



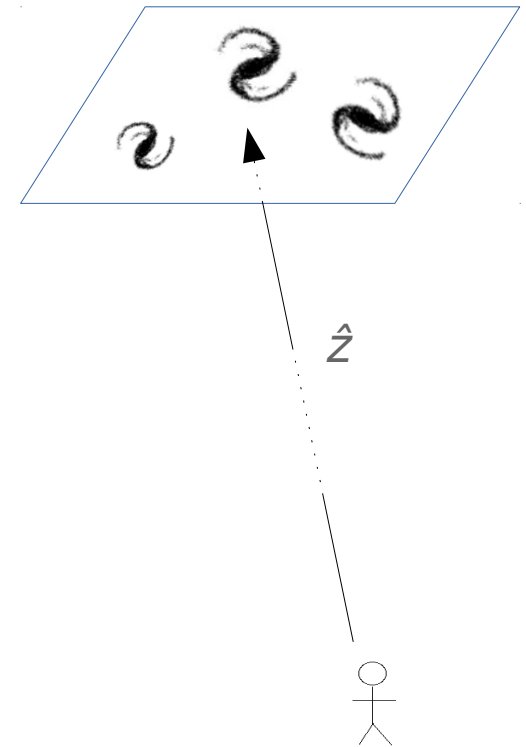
Matter density contrast (Redshift space)

Fourier space

$$\delta_s(\vec{k}) = \int d^3\vec{x} e^{-i\vec{k}\cdot\vec{x}} \left[e^{-i\vec{k}\cdot\vec{x}(\vec{x}\cdot\vec{v})} - 1 \right] [1 + \delta(\vec{x})]$$

Distant observer approximation

$$\begin{aligned} \delta_s(\vec{k}) = & \delta(\vec{k}) + \frac{k_z^2}{k^2} \frac{\dot{\delta}(\vec{k})}{H} - \epsilon^{zij} \frac{k_z k_i}{k^2} \frac{\pi_{(v)j}(\vec{k})}{H} \\ & + \frac{i^2}{2} \left(\frac{k_z}{aH} \right)^2 [v_z^2]_{\vec{k}} - \frac{i^3}{3} \left(\frac{k_z}{aH} \right)^3 [v_z^3]_{\vec{k}} \\ & + \frac{i^2}{2} \left(\frac{k_z}{aH} \right)^2 [\delta v_z^2]_{\vec{k}}. \end{aligned}$$



v_z Projection along l.o.s.

$$v^i(\vec{x}, t) = \frac{\pi^i(\vec{x}, t)}{\rho(\vec{x}, t)}$$

IV. EFToLSS in Redshift Space-

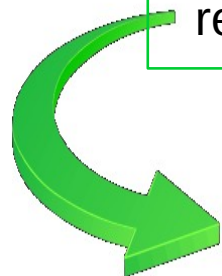
- 1) The idea
- 2) 1-loop matter power spectrum
- 3) Divergences

The idea...

- Redshift Space Distortions

$$\boxed{\begin{array}{c} \delta \\ \text{redshift space} \end{array}} = \boxed{\begin{array}{c} \delta \\ \text{real space} \end{array}} + \boxed{\begin{array}{c} \mathbf{V} \\ \text{operators} \end{array}}$$

- Renormalised density contrast


$$\boxed{\begin{array}{c} \text{Renormalised} \\ \delta \\ \text{redshift space} \end{array}} = \boxed{\begin{array}{c} \delta \\ \text{real space} + \\ \text{COUNTERTERMS} \end{array}} + \boxed{\begin{array}{c} \text{Renormalised} \\ \mathbf{V} \text{ operators} \end{array}}$$

EFT of LSS
(real space)

$P_{\delta\delta|1\text{loop}}^s$

?

Renormalised composite operators

- **Diagrams:** bare operator + UV information

$$[v_z^2]_{\vec{k}}^R = [v_z^2]_{\vec{k}} + \left(\frac{aH}{k_{NL}} \right)^2 \left[\boxed{c_1} + \left(\boxed{c_2} + \boxed{c_3} \frac{k_z^2}{k^2} \right) \delta(\vec{k}) \right]$$

$$[v_z^3]_{\vec{k}}^R = [v_z^3]_{\vec{k}} + \left(\frac{aH}{k_{NL}} \right)^2 3 \boxed{c_1} v_z(\vec{k})$$

$$[\delta v_z^2]_{\vec{k}}^R = [\delta v_z^2]_{\vec{k}} + \left(\frac{aH}{k_{NL}} \right)^2 \boxed{c_1} \delta(\vec{k})$$

R → renormalised
 $[\]_{\vec{k}} \rightarrow$ convolution

1-loop matter power spectrum in Redshift Space

Senatore, Zaldarriaga 2014

Real space

$$\begin{aligned}
 P_{\delta\delta|1\text{loop}}^s(k, \mu, z) = & \boxed{P_{\delta\delta|1\text{loop}}(k, z)} + 2\mu^2 \boxed{P_{\delta \frac{\dot{\delta}}{H}|1\text{loop}}(k, z)} + \mu^4 \boxed{P_{\frac{\dot{\delta}}{H} \frac{\dot{\delta}}{H}|1\text{loop}}(k, z)} \\
 & - \left(\frac{k\mu}{aH}\right)^2 P_{\delta[v_z^2]| \text{Tree}}(k, z) - \mu^2 \left(\frac{k\mu}{aH}\right)^2 P_{\frac{\dot{\delta}}{H}[v_z^2]| \text{Tree}}(k, z) \\
 & + \frac{1}{4} \left(\frac{k\mu}{aH}\right)^4 P_{[v_z^2][v_z^2]| \text{Tree}}(k, z) + (1 + f\mu^2) \left(\frac{k\mu}{aH}\right)^2 P_{\delta[\delta v_z^2]| \text{Tree}}(k, z) \\
 & + \frac{i}{3} (1 + f\mu^2) \left(\frac{k\mu}{aH}\right)^3 P_{\delta[v_z^3]| \text{Tree}}(k, z) \\
 & - (1 + f\mu^2) \left[\boxed{(c_1 + c_2)\mu^2} + \boxed{(\tilde{c}_1 + c_3)\mu^4} \right] \left(\frac{k}{k_{NL}}\right)^2 P_{\delta\delta|11}(k, z)
 \end{aligned}$$

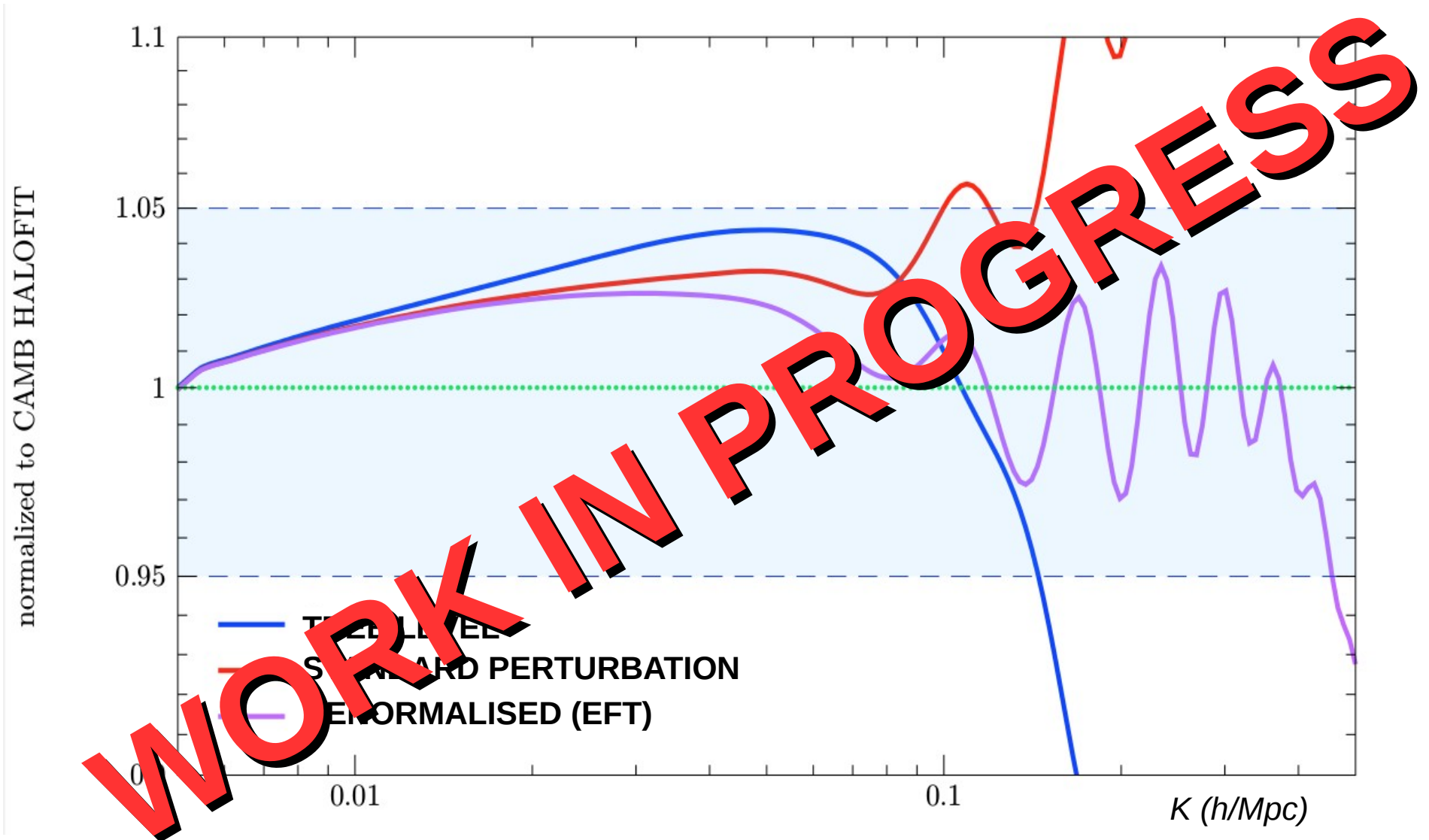
$$\mu \equiv k_z/k.$$

$$v_z \text{ Projection along l.o.s.}$$

$$f = \frac{d \ln \delta}{d \ln a}$$

$$\tilde{c}_1 \equiv c_1 f / aH$$

1 LOOP MATTER POWER SPECTRUM in REDSHIFT SPACE



Conclusions & future work

CONCLUSIONS

- The Universe is treated as a fluid. Most of the relevant information in Cosmology is found at **large scales**.
- At large scales, galaxies are point-like objects. There exist voids, filaments, clusters of galaxies...
- Large scale properties have memory of short distance physics.
- **Effective Field Theory of Large Scale Structures**
 - encodes small scale effects in the so called **counterterms** (deviations from perfect fluid).
 - Probes scales of interest where **Standard Perturbation Theory** breaks down.
- Adding **Redshift Space Distortion** effects allows us to learn about **peculiar velocities** within clusters.

& PROSPECTS

- To obtain the renormalisation parameters for the 1 loop matter power spectrum in Redshift Space.
- To study possible IR divergences.
- Compare with observations and N-body simulations.
- To apply this tool to the analysis of the **screening mechanism** in theories of **Modified Gravity** in collaboration with L. Perenon & C. Marinoni.



...1-loop $P_{\delta\delta}$ renormalisation

- $P_{\delta\delta|1\text{-loop}} = P_{11} + P_{13} + P_{\text{CT}}$


 Tree level UV-div

- Low-k behaviour (analytic terms) \rightarrow Taylor expansion loop integrals

$$P_{13}(k, z) \approx P_{11}(k, z) k^2 h(z) \underbrace{\int_0^\Lambda \frac{d^d q}{2\pi^2} \mathcal{P}_{\mathcal{R}}(q)}_{\mathcal{A}(\Lambda)}$$

Therefore,

$$P_{\delta\delta|1\text{-loop}} = P_{11} (1 + c_s^2 h(z) k^2)$$

Cutoff dependence
 eliminated by CT in
 the UV limit
 (same k & z dependence
 up to a constant)


Renormalisation parameter
 Fixed observationally or by simulations