STRUCTURE FORMATION CONSTRAINTS IN DARK ENERGY AND MODIFIED GRAVITY THEORIES WITHIN THE EFFECTIVE FIELD THEORY FORMALISM

Master II Thesis

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Outline

1. Motivation
2. Cosmological perturbations in General Relativity
3. Effective Field Theory formalism
4. Strategy
5. Theories under analysis
   - ΛCDM
   - ωCDM
   - Quintessence
     - Inverse Power-Law
     - Double Exponential Potential
   - Jordan Fierz Brans Dicke
6. Results and Conclusions

The goal :

To find the model which best explains the current accelerating phase of the Universe by using growth rate data.

1. We study the growth structure in Dark Energy (DE) and Modified Gravity (MG) theories : **Quintessence**, ω**CDM** and **Jordan Fierz Brans Dicke** (JFBD).

2. We use galaxy power spectra observational data Table 2 to constrain the free parameters of our theories.

Survey	Redshift, z	$f\sigma_8(z)$	Reference
THF	0.02	0.40 ± 0.07	[1]
DNM	0.02	0.31 ± 0.05	[2]
6dFGS	0.07	0.42 ± 0.06	[3]
2dFGRS	0.17	0.42 ± 0.06	[4, 5]
2SLAQ	0.55	0.45 ± 0.05	[6]
SDSS LRG	0.34	0.53 ± 0.07	[7, 8]
	0.25	0.35 ± 0.06	
	0.37	0.46 ± 0.04	
BOSS	0.57	0.43 ± 0.07	[9]
WiggleZ	0.20	0.40 ± 0.13	[10]
	0.40	0.39 ± 0.08	
	0.60	0.40 ± 0.07	
	0.76	0.48 ± 0.09	
VVDS	0.77	0.49 ± 0.18	[5, 11]
VIPERS	0.80	0.47 ± 0.08	[12]

3. χ^2 analysis to evaluate the best-fitting model and compare the results with the Cosmological Concordance Model, ΛCDM.

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Motivation

DE and MG theories appear as an alternative solution to some theoretical and phenomenological issues in General Relativity (GR).

These theories must

- preserve success of ΛCDM in previous Cosmological epochs,
- allow the formation of structures of the Universe nowadays,
- drive accelerating expansion of the Universe today.

How to distinguish among DE and MG theories?

- Growth structure observations are sensitive to both **background evolution** and **cosmological linear matter density perturbations**.
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V. Mukhanov, Cambridge University Press.18 (2005).

- The matter density perturbation equation. Density contrast $\delta = \frac{\rho - \rho_0}{\rho_0}$.

1. RW metric in *longitudinal gauge*

$$ds^2 = a^2(\eta)\{(1+2\Phi)d\eta^2 - (1-2\Psi)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]\}, \quad (1)$$

Φ and Ψ Bardeen's potentials.

2. We obtain the perturbed equations of motions up to linear order :

$$\delta G^\mu_\nu = -8\pi G \delta T^\mu_\nu. \quad (2)$$

3. We assume :
   - Perfect fluid behavior.
   - Adiabatic perturbations (entropy is constant).
   - Quasi-static approximation (QSA). Time derivatives are small with respect to spatial derivatives.

4. Fourier space :

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G \rho_m(t)\delta = 0. \quad (3)$$

Valid for sub-Hubble modes, $k >> H$.

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- The growth structure function is defined as $f(z)\sigma_{0,8}\delta(z)$, being $f(z) = \frac{d\ln\delta}{d\ln a}$ the growth rate and $\sigma_{0,8} \equiv 0.8$.

Effective Field Theory formalism

EFT FORMALISM

G. Gubitosi, F. Piazza and F. Vernizzi, JCAP **1302** (2013) 032 [arXiv :1210.0201 [hep-th]].

- The action of many theories can be written (in *unitary gauge*) in function of the so-called structural functions : $M(t)$, $\lambda(t)$, $C(t)$, $\mu_2^2(t)$, $\mu_3(t)$ and $\epsilon_4(t)$.

Theory	$\mu = \frac{dlog(M^2(t))}{dt}$	$\lambda(t)$	$C(t)$	$\mu_2^2(t)$	$\mu_3(t)$	$\epsilon_4(t)$
ΛCDM	0	*const.*	0	0	0	0
ωCDM	0	✓	0	0	0	0
Quintessence	0	✓	✓	0	0	0
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- Density perturbation equation can be derived (analogously to GR) :

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being $\Omega_m(t)$ the matter content and

$$G_{eff} = \frac{1}{G} \frac{1}{8\pi M^2(1+\epsilon_4)} \frac{2C + \tilde{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\tilde{\epsilon}_4 + 2(\mu + \tilde{\epsilon}_4)^2 + Y_{IR}(t,k)}{2C + \tilde{\mu}_3 - 2\dot{H}\epsilon_4 + 2H\tilde{\epsilon}_4 + 2\frac{(\mu+\tilde{\epsilon}_4)(\mu-\mu_3)}{1+\epsilon_4} - \frac{(\mu-\mu_3)^2}{2(1+\epsilon_4)^2} + Y_{IR}(t,k)} \quad (5)$$

where $\tilde{\mu}_3$, $\tilde{\epsilon}_4$ and $Y_{IR}(t,k)$ are combinations of the structural functions.

- Stability conditions :

$$A = (C + 2\mu_2^2)(1+\epsilon_4) + \frac{3}{4}(\mu - \mu_3) \geq 0, \quad \text{Ghost free ✓} \quad (6)$$

$$B = (C + \tilde{\mu}_3/2 - \dot{H}\epsilon_4 + H\tilde{\epsilon}_4)(1+\epsilon_4) - (\mu - \mu_3)\left(\frac{\mu - \mu_3}{4(1+\epsilon_4)} - \mu - \tilde{\epsilon}_4\right) \geq 0. \quad \text{Gradient stability ✓} \quad (7)$$

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Strategy

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4D spatially flat, homogeneous and isotropic Universe in expansion described by the Robertson Walker (RW) metric in comoving coordinates

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]. \quad (8)$$

1. Theories written within the **Effective Field Theory** (EFT) formalism
   - it allows to gather all of them in a very compact manner,
   - it provides a neat separation between background and perturbation sectors
   - and it permits to have under control any kind of instability.
2. The background equations are solved.
3. We study the growth structure function assuming that the theories behave as ΛCDM at the present time.
4. χ^2 analysis constrains the free parameters of our theories.
5. In order to compare theories with different number of free parameters, κ, the reduced χ^2 is computed

$$\chi^2_{red} = \chi^2_{min}/\nu \quad (9)$$

being $\nu = N - \kappa - 1$ the number of degrees of freedom of the theory and $N = \#$ data points.

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Theories under analysis

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- Action : $S_{\Lambda CDM} = \int d^4x\sqrt{|g|}\frac{M_{Pl}^2}{2}\left[R - 2\Lambda\right]$.
- Exact solution of the Friedmann equation for flat Universe with matter and cosmological constant (c.c) :

$$a(t) = \left(\frac{\Omega_{m,0}}{\Omega_{\Lambda,0}}\right)\left[\sinh\left(\frac{3\sqrt{\Omega_{\Lambda,0}}}{2}H_0 t\right)\right]^{2/3} \tag{10}$$

  being $\Omega_{m,0}$ and $\Omega_{\Lambda,0}$ the matter and the c.c content at the present time, respectively, and H_0 is the Hubble parameter today.
- Perturbation equation.

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_m\delta = 0, \tag{11}$$

  Initial conditions (I.C.) are set in the past ($z = 1000$) when $\delta(t) \simeq a(t)$.
- Growth structure function is computed.
- χ^2 analysis : $\chi^2_{min} = 11.689$ ($\chi^2_{red} = 0.899$) at $\Omega_{m,0} = 0.302$ and $\Omega_{\Lambda,0} = 0.698$.

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ωCDM

ωCDM

- Characterized by a constant equation of state parameter, $\omega \equiv const$. ΛCDM belongs to this kind of theories.

- The Friedmann equation reads as

$$\left(\frac{a'(\tau)}{a(\tau)}\right)^2 = \Omega_{m,0}a(\tau)^{-3} + \Omega_{\Lambda,0}a(\tau)^{-3(1+\omega)}, \tag{12}$$

being prime the derivative w.r.t. the dimensionless time, $\tau = H_0 t$.

- Matter density perturbation equation, expression (4) :

$$\delta'' + 2\frac{H}{H_0}\delta' - \frac{3}{2}G_{eff}\Omega_m(\tau)\delta = 0, \tag{13}$$

being $G_{eff} = 1$, particularizing (5). The I.C. are imposed today $\delta(\tau_{today}) = \delta_{\Lambda CDM}(\tau_{today})$ and $\delta'(\tau_{today}) = \delta'_{\Lambda CDM}(\tau_{today})$.

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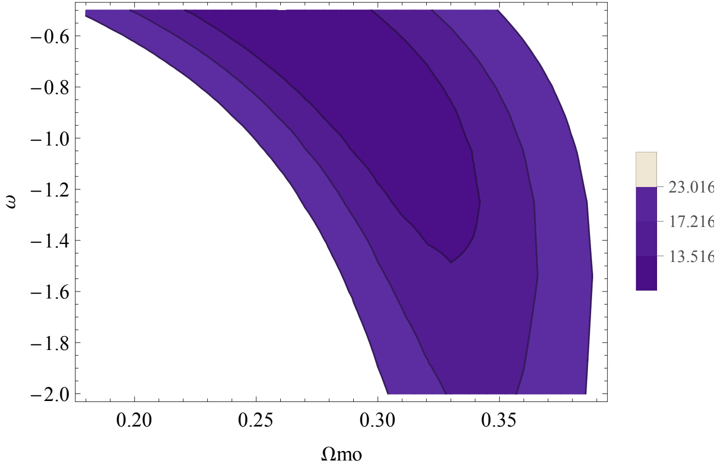
ωCDM results.

Parameters $\{\Omega_{m,0}, \omega\}$.



Result of the χ^2 test for ωCDM models. Confidence levels 68%, 95% and 99% are plotted.

- The minimum $\chi^2_{min} = 11.216$ ($\chi^2_{red} = 0.935$) is found at $\Omega_{m,0} = 0.27$, $\omega = -0.59$.
- ΛCDM, $\omega = -1$, is not the best-fitting model of ωCDM theory.

Quintessence

Quintessence

S. Tsujikawa, Class. Quant. Grav. **30** (2013) 214003 [arXiv :1304.1961 [gr-qc]].

- Action : $S_\phi = \int d^4x\sqrt{|g|}\left[\frac{M_{Pl}^2}{2}R + \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) - V(\phi)\right]$.
- Background and field equations :

$$\left(\frac{a'(\tau)}{a(\tau)}\right)^2 = \frac{\tilde{\phi}'^2}{6} + \tilde{V}(\tilde{\phi}) + \Omega_{m,0}a^{-3}(\tau), \quad (14)$$

$$\tilde{\phi}'' + 3\frac{a'}{a}\tilde{\phi}' + \frac{\partial\tilde{V}(\tilde{\phi})}{\partial\tilde{\phi}} = 0 \quad (15)$$

  where prime is the dimensionless time derivative, $\tilde{\phi} = \sqrt{8\pi G}\phi$ and $\tilde{V}(\tilde{\phi})$ is the dimensionless potential.
  The I.C. are imposed in the past, $a(0) = 10^{-3}$, $\phi(0) = 0.5$ and its time derivative $\phi'(0) = 0$ (G. Barro Calvo and A. L. Maroto, Phys. Rev. D **74** (2006) 083519 [astro-ph/0604409]).
- Matter density perturbation (4). $G_{eff} = 1$.
  I.C. $\delta(\tau_{today}) = \delta_{\Lambda CDM}(\tau_{today})$ and $\delta'(\tau_{today}) = \delta'_{\Lambda CDM}(\tau_{today})$.
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- Models : **Inverse Power-Law** (IPL) and **Double Exponential Potential** (2EP).
- The growth structure function is computed and the χ^2 analysis implemented.

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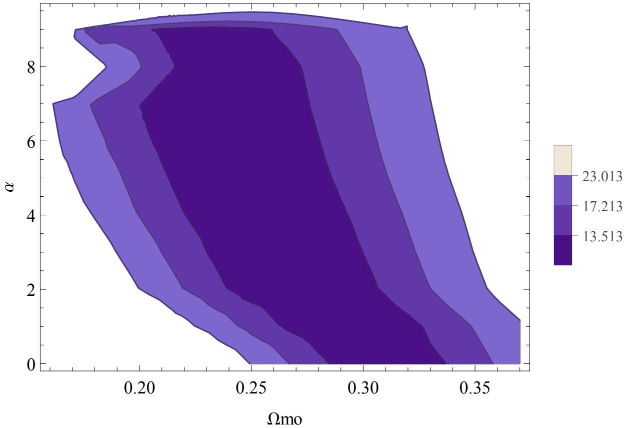
- Models : **Inverse Power-Law** (IPL) and **Double Exponential Potential** (2EP).
- The growth structure function is computed and the χ^2 analysis implemented.

IPL result.

Potential : $\tilde{V}(\tilde{\phi}) = A/\tilde{\phi}^{\alpha}$, parameters $\{\Omega_{m,0}, \alpha\}$. $A = A(\Omega_{m,0}, \alpha)$.

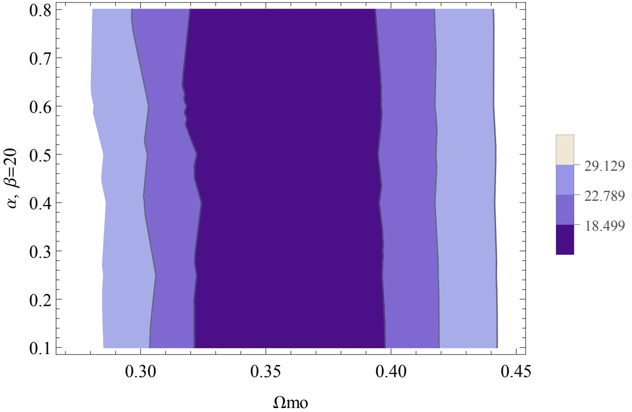


Result of the χ^2 test for IPL models. Confidence levels 68%, 95% and 99% are plotted.

- The minimum $\chi^2_{min} = 11.213$ ($\chi^2_{red} = 0.934$) is found at $\Omega_{m,0} = 0.265$, $\alpha = 3$ and $A = 8.032$.
- α cannot take negative values since the potential would stop being an inverse power-law.

2EP result.

Potential : $\tilde{V}(\tilde{\phi}) = A(e^{\alpha\tilde{\phi}} + e^{\beta\tilde{\phi}})$, parameters $\{\Omega_{m,0}, \alpha, \beta\}$. $A = A(\Omega_{m,0}, \alpha)$.



Result of the χ^2 test for 2EP models. C.L. 68%, 95% and 99%.

- $0 < \alpha < 0.8$ since $\omega_\phi < -0.8$ and $\beta > 5.5$ due to Nucleosynthesis constraints, T. Barreiro, E. J. Copeland and N. J. Nunes, Phys. Rev. D **61** (2000) 127301 [astro-ph/9910214].
- $\beta = 20$ G. Barro Calvo and A. L. Maroto, Phys. Rev. D **74** (2006) 083519 [astro-ph/0604409].
- $\chi^2_{min} = 11.216$ ($\chi^2_{red} = 1.361$) is found at $\Omega_{m,0} = 0.359$, $\beta = 20$, $\alpha = 0.6$ and $A = 0.797$.

Jordan Fierz Brans Dicke

JFBD

B. Boisseau, G. Esposito-Farese, D. Polarski and A. A. Starobinsky, Phys. Rev. Lett. **85** (2000) 2236 [gr-qc/0001066].

- The action in the Jordan Frame reads as

$$S_{JFBD} = \int d^4x \frac{\sqrt{|g|}}{16\pi G}\left(\phi R + \frac{\omega_{BD}}{\phi} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right),$$

being $\omega_{BD} = -\frac{3-1/\alpha^2}{2}$ the characteristic parameter of JFBD theories.

- Background evolution. The contribution of ϕ to $H(t)$ is quite negligible, $\phi'^2/H \sim 10^{-11} - 10^{-17}$. Then, the solution is $a(t) \simeq a_{\Lambda\mathrm{CDM}}(t)$, see (10).
- Density perturbation equation (4). G_{eff} (5) is function of time and Fourier modes, k,

$$G_{eff} = \frac{1}{G}\frac{1}{8\pi M^2}\frac{2C + 2\mu^2 + Y_{IR}(t,k)}{2C + \frac{3}{2}\mu^2 + Y_{IR}(t,k)} \quad (16)$$

but we assume $k \gtrsim 100\mathrm{hMpc}^{-1}$. Hence the dependence of G_{eff} on the k modes is negligible.

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- Density perturbation equation (4). G_{eff} (5) is function of time and Fourier modes, k,

$$G_{eff} = \frac{1}{G}\frac{1}{8\pi M^2}\frac{2C + 2\mu^2 + Y_{IR}(t,k)}{2C + \frac{3}{2}\mu^2 + Y_{IR}(t,k)} \quad (16)$$

but we assume $k \gtrsim 100\mathrm{hMpc}^{-1}$. Hence the dependence of G_{eff} on the k modes is negligible.

- Stable theory, conditions (6) and (7) are satisfied.
- The growth structure function is computed and the χ^2 analysis implemented.

JFBD

B. Boisseau, G. Esposito-Farese, D. Polarski and A. A. Starobinsky, Phys. Rev. Lett. **85** (2000) 2236 [gr-qc/0001066].

- The action in the Jordan Frame reads as

$$S_{JFBD} = \int d^4x \frac{\sqrt{|g|}}{16\pi G}\left(\phi R + \frac{\omega_{BD}}{\phi} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right),$$

  being $\omega_{BD} = -\frac{3-1/\alpha^2}{2}$ the characteristic parameter of JFBD theories.
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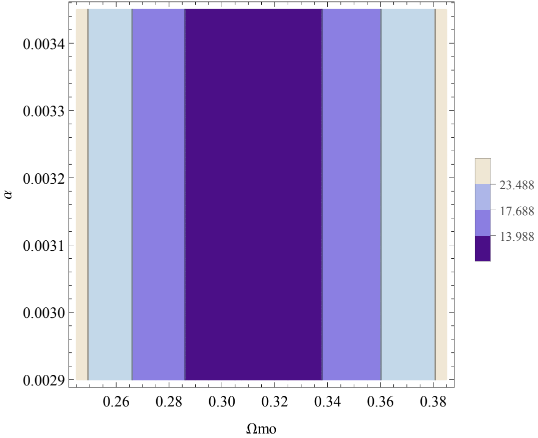
JFBD results.

Potential : $V(\phi) = 0$, parameters $\{\Omega_{m,0}, \alpha\}$. $\omega_{BD} = -\frac{3-1/\alpha^2}{2}$.



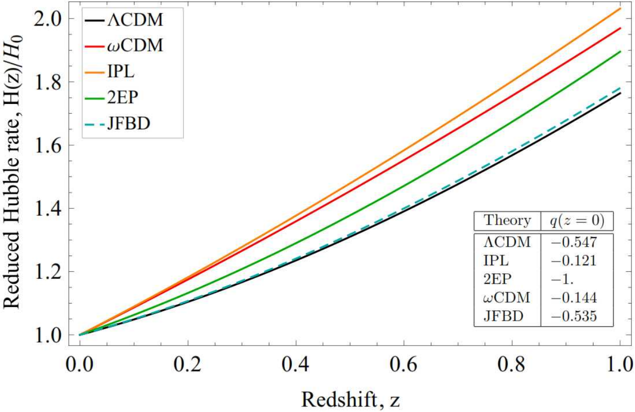
Result of the χ^2 test for JFBD models. C.L. 68%, 95% and 99%.

- Cassini Mission, B. Bertotti, L. Iess and P. Tortora, Nature 425,374-376 limits $\alpha < 3.45 \cdot 10^{-3}$.
- $\chi^2_{min} = 11.6884$ ($\chi^2_{red} = 0.974$) is found at $\Omega_{m,0} = 0.31$, $\alpha = 3.00 \cdot 10^{-3}$ and $\omega_{BD} = 5.5 \cdot 10^4$.

Results and Conclusions

Results

Background evolution of the best-fitting models.



Reduced Hubble rate evolution. All the models present a similar tendency.

- The reduced Hubble parameter today is equal to one, $1 = \Omega_{m,0} + \Omega_{\phi,0}$.
- Every model causes acceleration, $q(z_0) = -\frac{\ddot{a}_0}{a_0 H_0^2} < 0$, but ΛCDM is the one closer to the observed value $q_{obs}(0) = -0.55$.

Results

Growth structure of the best-fitting models.

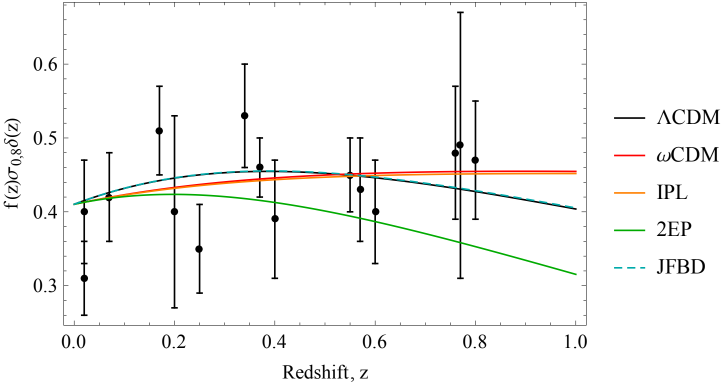
Theory	χ^2_{red}	$\Omega_{m,0}$	Second parameter	Third parameter
ΛCDM	0.899	$0.302^{+0.068}_{-0.012}$	$\omega = -1$	–
IPL	0.934	$0.265^{+0.075}_{-0.065}$	$\alpha = 3^{+7}_{-3}$	$A = 8.032^{+10^8}_{-7.257}$
2EP	1.361	$0.359^{+0.010}_{-0.049}$	$\alpha = 0.6^{+0.2}_{-0.5}$, $\beta = 20$	$A = 0.797^{+0.010}_{-0.096}$
ωCDM	0.935	$0.270^{+0.070}_{-0.070}$	$\omega = -0.59^{+0.09}_{-0.91}$	–
JFBD	0.974	$0.310^{+0.030}_{-0.030}$	$\alpha = (3.00 \cdot 10^{-3})^{+0.00345}_{-0.00010}$	$\omega_{BD} = (5.5 \cdot 10^4)^{+6.0\cdot 10^4}_{-1.3\cdot 10^4}$

The table presents every model, their reduced χ^2 value and the best range of parameters (C.L. 68%).

- χ^2_{red} is worse for models with larger number of free parameters.
- The best-fitting model is ΛCDM (lowest $\chi^2_{red} = 0.899$) followed by IPL.

Results

Growth structure of the best-fitting models.



Growth structure function and datapoints Table 2.

- All the models show the same value as $\Lambda\mathrm{CDM}$ at the present time, as we imposed by assumption, but they deviate in the past.
- JFBD curve very close to $\Lambda\mathrm{CDM}$ but it is not the second best-fitting model.
- 2EP : the most deviated model and the worst fitting.

Conclusions

1. We computed the growth structure function for ωCDM, Quintessence and Jordan Fierz Brans Dicke theories.
2. We solved the background and perturbation equations by assuming that initially theories behave as ΛCDM.
3. A χ^2 analysis is implemented by using galaxy power spectra data Table 2.
4. We realized that ΛCDM is not the best-fitting model of ωCDM theory.
5. Results show that ΛCDM is the best-fitting model with $\Omega_{m,0} = 0.302$ followed by the Inverse Power-Law model of Quintessence $\{\Omega_{m,0} = 0.265, \alpha = 3\}$.

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Prospects

Some parameters of the theories are not fully constrained. Possible ways of extending our results :

- To use other kind of observational data, such as supernovae datasets G. Barro Calvo and A. L. Maroto, Phys. Rev. D **74** (2006) 083519 [astro-ph/0604409] and to overlap their contourplots in order to limit the bound of the involved parameters.
- To study their power spectra. Quasi-static approximation must be dropped out, otherwise the transfer function and consequently the power spectrum would be flat.
- To probe new range of redshifts and to check whether $\Lambda\mathrm{CDM}$ keep being the best-fitting model.
- To analyze Jordan Fierz Brans Dicke models with arbitrary potentials.
- To sweep further values of β (Double Exponential Potential model of Quintessence) and to implement a marginalized χ^2 test. ■

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