The Effective Field Theory of Large-Scale Structure

Themed Discussion
(August 28, 17:00 UTC)

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Standard Perturbation Theory

- Treat matter as a **perfect fluid** obeying the fluid equations

$\dot{\delta} + \nabla \cdot [(1+\delta)\boldsymbol{v}] = 0,$	Continuity Equation
$\dot{\boldsymbol{v}} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\mathcal{H}\boldsymbol{v} - \nabla\phi,$	Euler Equation
$\nabla^2\phi = 4\pi G a^2 \bar{\rho}\delta,$	Poisson Equation

- Solve the equations perturbatively in powers of the **density field**, **δ**

$$\delta(\mathbf{k}, \tau) = D(\tau)\delta^{(1)}(\mathbf{k}) + D^2(\tau)\delta^{(2)}(\mathbf{k}) + D^3(\tau)\delta^{(3)}(\mathbf{k}) + \ldots$$

Growth factor

2nd order density field

e.g. Bernardeau+02

Standard Perturbation Theory

- The solution is a **linear power spectrum** plus **loop corrections**

$$P(\mathbf{k},\tau) = P_L(\mathbf{k},\tau) + P_{22}(\mathbf{k},\tau) + 2P_{13}(\mathbf{k},\tau) + \dots$$

Linear *One-loop*

- Loop corrections are **integrals** over powers of the linear power spectrum

$$\begin{aligned} P_{22}(\mathbf{k}) &= \int \frac{d^3q}{(2\pi)^3} P_L(\mathbf{q}) P_L(\mathbf{k}-\mathbf{q}) |F_2(\mathbf{q},\mathbf{k}-\mathbf{q})|^2 \\ P_{13}(\mathbf{k}) &= 3P_L(\mathbf{k}) \int \frac{d^3q}{(2\pi)^3} P_L(\mathbf{q}) F_3(\mathbf{k},\mathbf{q},-\mathbf{q}) \end{aligned}$$

Gravitational Kernels

e.g. Bernardeau+02

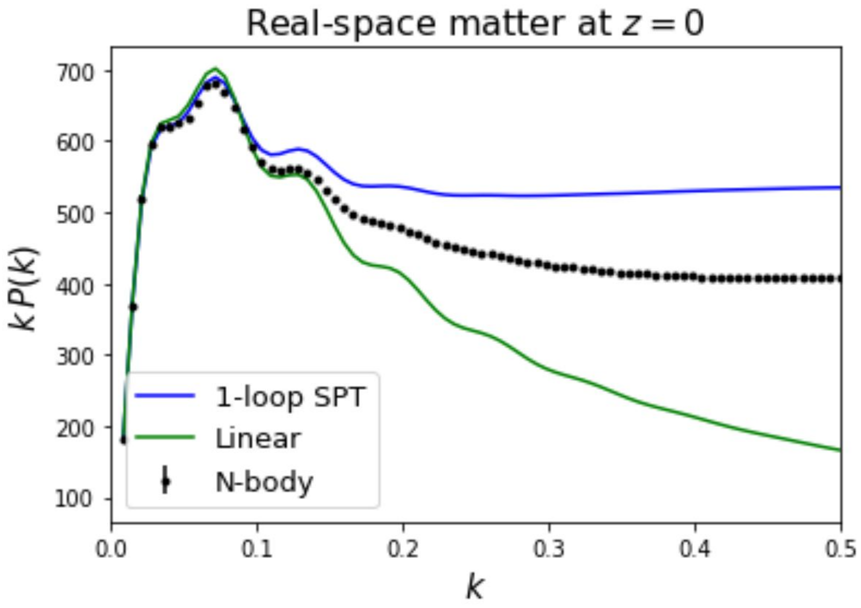
Standard Perturbation Theory

Why does SPT fail?

1. The expansion parameter, $\boldsymbol{\delta}$, is **not small**

2. Matter is not a **perfect fluid**

3. The loop integrals can **diverge**
   No UV - protection



Using Quijote simulations (Villaescusa-Navarro+19) and CLASS-PT (Ivanov+20)

Effective Field Theory

- Treat matter as an **imperfect** fluid, described by the Boltzmann equation, written in terms of the **smoothed** density field

$$\begin{aligned} \dot{\delta}_\Lambda + \nabla \cdot [(1+\delta_\Lambda \mathbf{v}_\Lambda] &= 0 \\ \dot{\mathbf{v}}_\Lambda + (\mathbf{v}_\Lambda \cdot \nabla)\,\mathbf{v}_\Lambda &= -\mathcal{H}\mathbf{v}_\Lambda - \nabla\phi_\Lambda \boxed{- \frac{1}{\rho_\Lambda}\nabla \underline{\underline{\tau}}} \end{aligned}$$

This adds a **stress-tensor, τ**, defined by:

$$\tau^{ji} = -\boxed{c_s^2}\rho\delta^{ij} + \boxed{\eta}\left(\partial^j v^i + \partial^i v^j\right) + \ldots$$

Sound-speed Viscosity

Baumann+10, Carrasco+12

Effective Field Theory

- At next-to-leading order:

$$P(k) = P_{\rm lin}(k) + P_{22,\Lambda}(k) + 2P_{13,\Lambda}(k) - 2c_{s,\Lambda}^2 k^2 P_{\rm lin}(k)$$

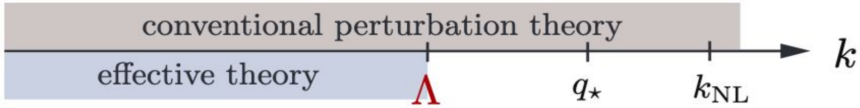
Counterterm

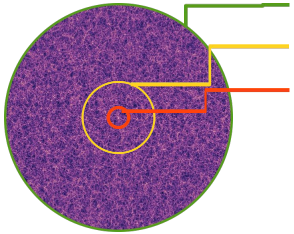
- The counterterm is the backreaction of **small-scale** physics on large-scale modes. It **must** be measured from simulations.
  - An analogy: the **viscosity** in fluid dynamics

Baumann+10, Carrasco+12

Effective Field Theory

- Equivalently, we can restrict to large-scales in **renormalization**





$$\xi(r) \longrightarrow P_{\delta\delta}(k) \supseteq \int_{k_{IR}}^{k*} \mathrm{d}^3\mathbf{q} f(\mathbf{q}) g(\mathbf{q}, \mathbf{k}-\mathbf{q}) + \int_{k*}^{k_{NL}} \mathrm{d}^3\mathbf{q} f(\mathbf{q}) g(\mathbf{q}, \mathbf{k}-\mathbf{q}) = P^{\rm SPT}_{1\ \rm loop}(k) - \underbrace{\frac{c_\delta^2}{k_{NL}^2}} k^2 P_{\rm lin}(k)$$

SPT works — Taylor expansion

COUNTER-TERM
Nbody simulations

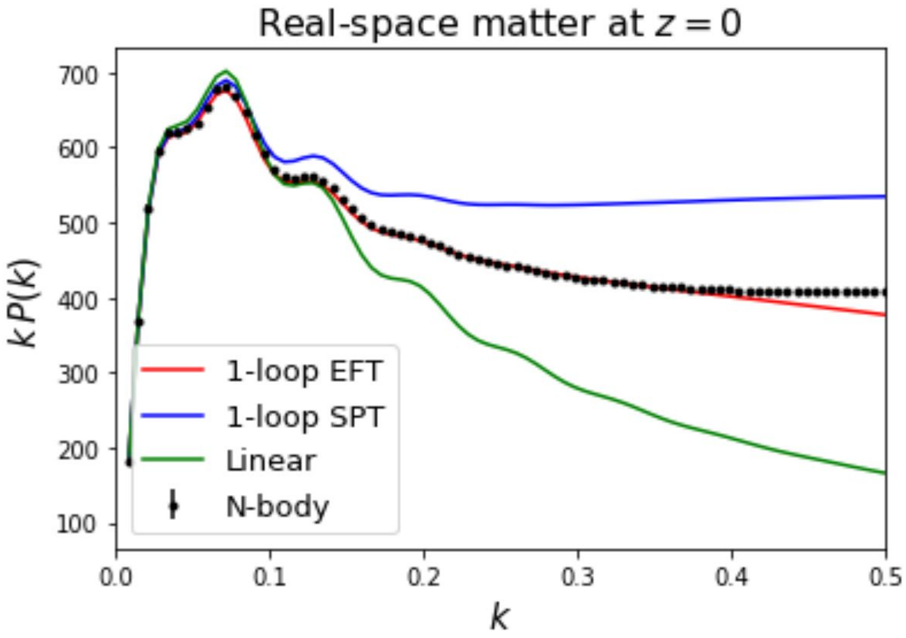
- The loop integrals seem to depend on the cut-off scale

  But this dependence is exactly captured by the **counterterm**!

Baumann+10, Carrasco+12

Effective Field Theory

- EFT provides a much better fit to data than SPT, since we include the non-ideal fluid contributions
- Extending to 2-loop order improves this further

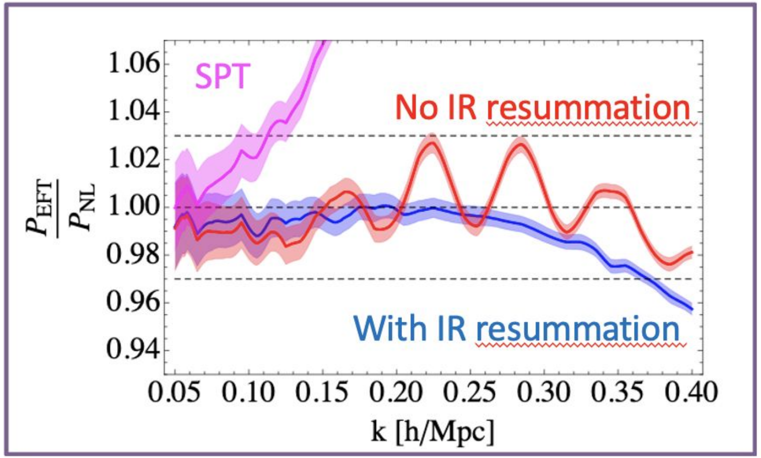


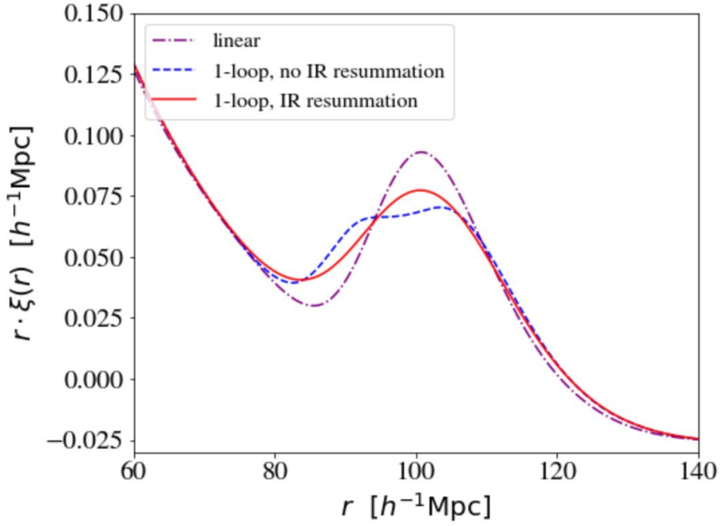
Using Quijote simulations (Villaescusa-Navarro+19) and CLASS-PT (Ivanov+20)

IR Resummation

- At late times, particles have **large displacements** that cannot be treated perturbatively.
- This **damps** the BAO wiggles
- The contributions can be **resummed**, e.g.

$$P_L(k) \to P_{nw}(k) + P_w(k)e^{-k^2\Sigma^2}$$





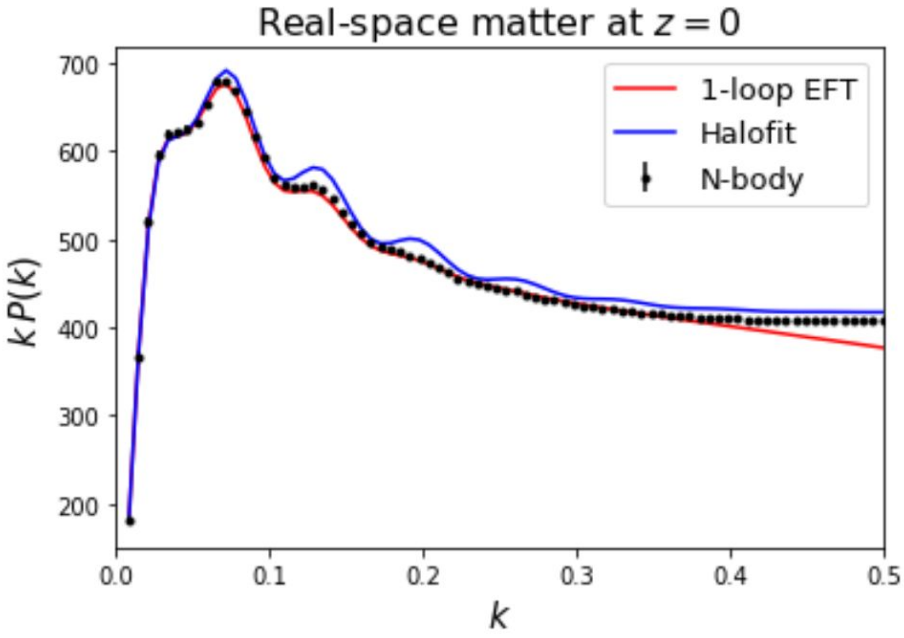
Comparison to HaloFit

HaloFit

- Calibrated from N-body simulations
- Extends further into non-linear regime

But

- Too wiggly!
- Only ~ 5% accurate



Using Quijote simulations (Villaescusa-Navarro+19) and CLASS-PT (Ivanov+20)

Biased Tracers

The simple approach: expand the galaxy overdensity in powers of δ:

$$\delta_g(\mathbf{x}) = b_1\delta(\mathbf{x}) + \tfrac{b_2}{2}\delta^2(\mathbf{x}) + \tfrac{b_3}{6}\delta^3(\mathbf{x}) + \ldots$$

The EFT approach: include all possible parameters allowed by symmetry

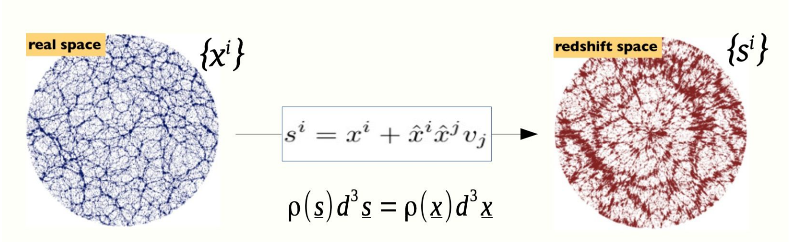
$$\delta_g = b_1\delta + \epsilon + \frac{b_2}{2}\delta^2 + b_{\mathcal{G}_2}\mathcal{G}_2 + \frac{b_3}{6}\delta^3 + b_{\delta\mathcal{G}_2}\delta\mathcal{G}_2 + b_{\mathcal{G}_3}\mathcal{G}_3 + b_{\Gamma_3}\Gamma_3 + R_*^2\partial^2\delta$$

with density operators, tidal operators, stochastic operators, and non-local operators (all integrated over a lightcone)

e.g. Ivanov+19, de la Bella+18

Redshift Space

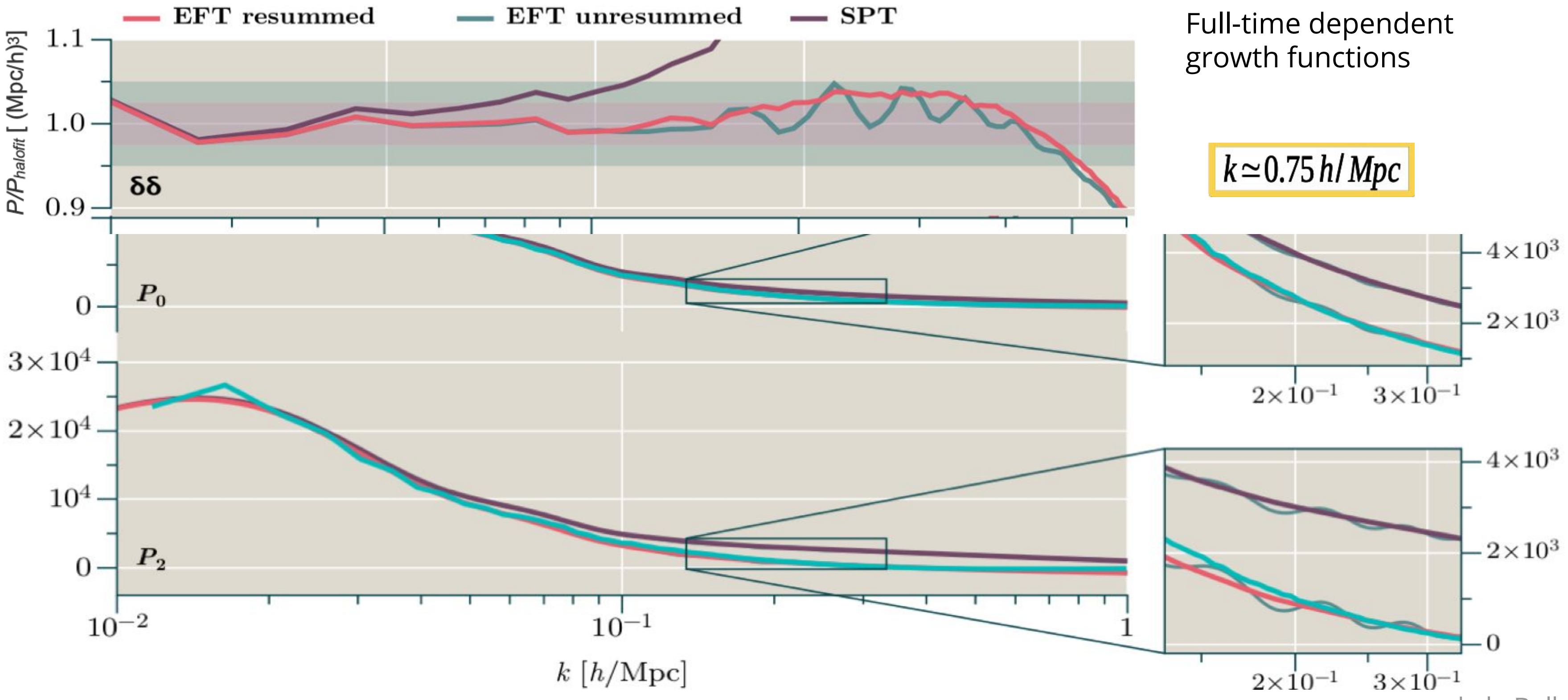
Galaxy surveys infer distances



$$\delta_r(\vec{k}) = \delta(\vec{k}) + \int d^3x \, e^{-i\vec{k}\cdot\vec{x}} \left(\exp\left[-i \frac{k_z}{aH} v_z(\vec{x}) \right] - 1 \right) (1 + \delta(\vec{x}))$$

Exact mapping. Smooth it → get new contributions (+2 at one-loop order)
Caveat: fingers - of - God - low k_NL

Redshift Space



Full-time dependent growth functions

$k \simeq 0.75\, h/Mpc$

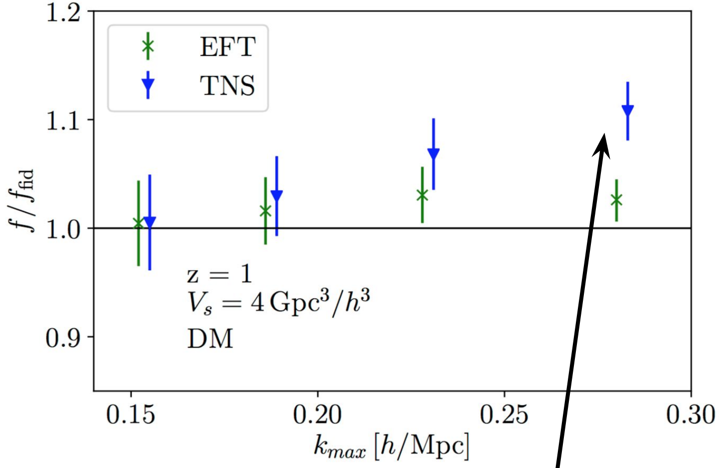
de la Bella+17

Comparison to TNS (~ halofit)

- Doesn't include corrections beyond perfect fluids
- Partly resums SPT contributions - does not help if the theory is wrong

- Doesn't capture the BAO - fits it only because of accidental shape of LCDM spectrum
- Doesn't capture fingers-of-God - large biases in redshift space for DM

- Cannot be precise more than ~3%
- Field level (no summary statistic) - stringent test. Only the linear part is OK



TNS gives a biased growth factor!

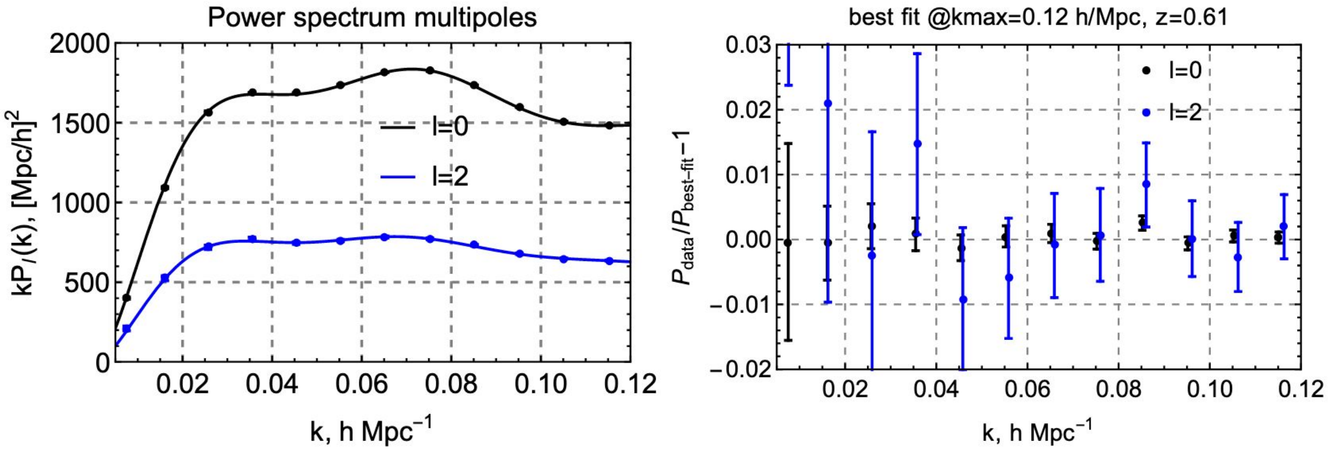
Taruya+10, Bose+19

The Full Model

The 1-loop model has free parameters:

$$\{b_1, b_2, b_{\mathcal{G}_2}, c_{s,0}, c_{s,2}, b_4, P_{\rm shot}\}$$

Sub-percent accurate in the mildly non-linear regime



Nishimichi+20

The Full Model

- Impact of ***bias*** & ***redshift-space*** *models* on the halo power spectrum.
- We develop **the advective bias model**.
- We use **EFT** to account for non-linear physics.

- **WizCOLA** simulation.
- Risk of **over-fitting**:
  - **Bayesian Information criterian**
  - Ensemble average.

	BIC	Min χ2/dof	Δχ2(%)
Linear+ KaiserTree	11.1	1.1	1.8
Linear+Kaiser Halo	11.1	1.0	3.1
Coev+Kaiser Halo	16.6	1.0	3.2
Coev+SPT	16.6	1.0	2.3
Coev+EFT	44.2	1.1	6.9
M&Roy+ KaiserTree	27.6	1.0	2.8
Advect+SPT	38.7	1.1	5.1
Advect+EFT	66.4	1.2	6.0

de la Bella+18

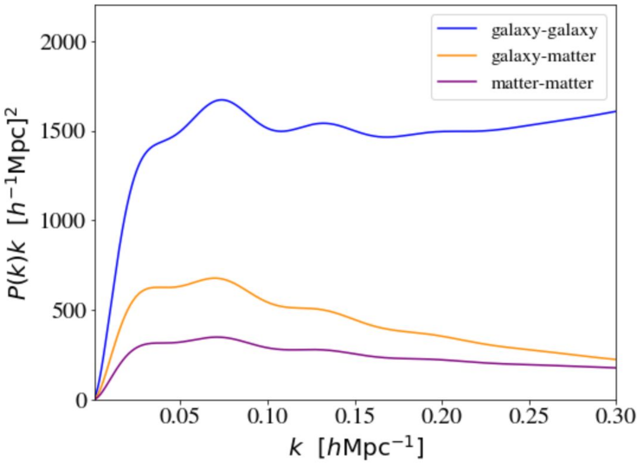
CLASS-PT

- Based on CLASS
- Computes the 1-loop PT integrals in < 1 s
- Includes all effects:
    - IR Resummation
    - Loop integrals
    - Biased tracers
    - Redshift-space distortions
    - Alcock-Pacyznski effects
- Can be interfaced with MontePython for MCMC sampling
- Tutorial - ?

```
# real space matter power spectrum
pk_full_ir = M1.pk_mm_real(cs)

# real space galaxy-galaxy power spectrum
pk_gg = M1.pk_gg_real(b1, b2, bG2, bGamma3, cs, cs0, Pshot)

# real space galaxy-matter power spectrum
pk_gm = M1.pk_gm_real(b1, b2, bG2, bGamma3, cs, cs0)
```



https://github.com/michalychforever/class-pt

Ivanov+19

Beyond LCDM

No problem to include beyond LCDM

- Any model that does not alter non-linear interactions - done
  (a) explicit time-dependence (very tiny effect)

- Modified gravity - worked out, but not included in the code yet (MG changes the non-linear interactions (`kernels’)