THE JOURNEY OF LIMBER & THE SkyPy PROJECT

Lucia F. de la Bella 09/02/2021

Who I am...

https://orcid.org/0000-0002-1064-3400
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Also known as

Lucia F. de la Bella

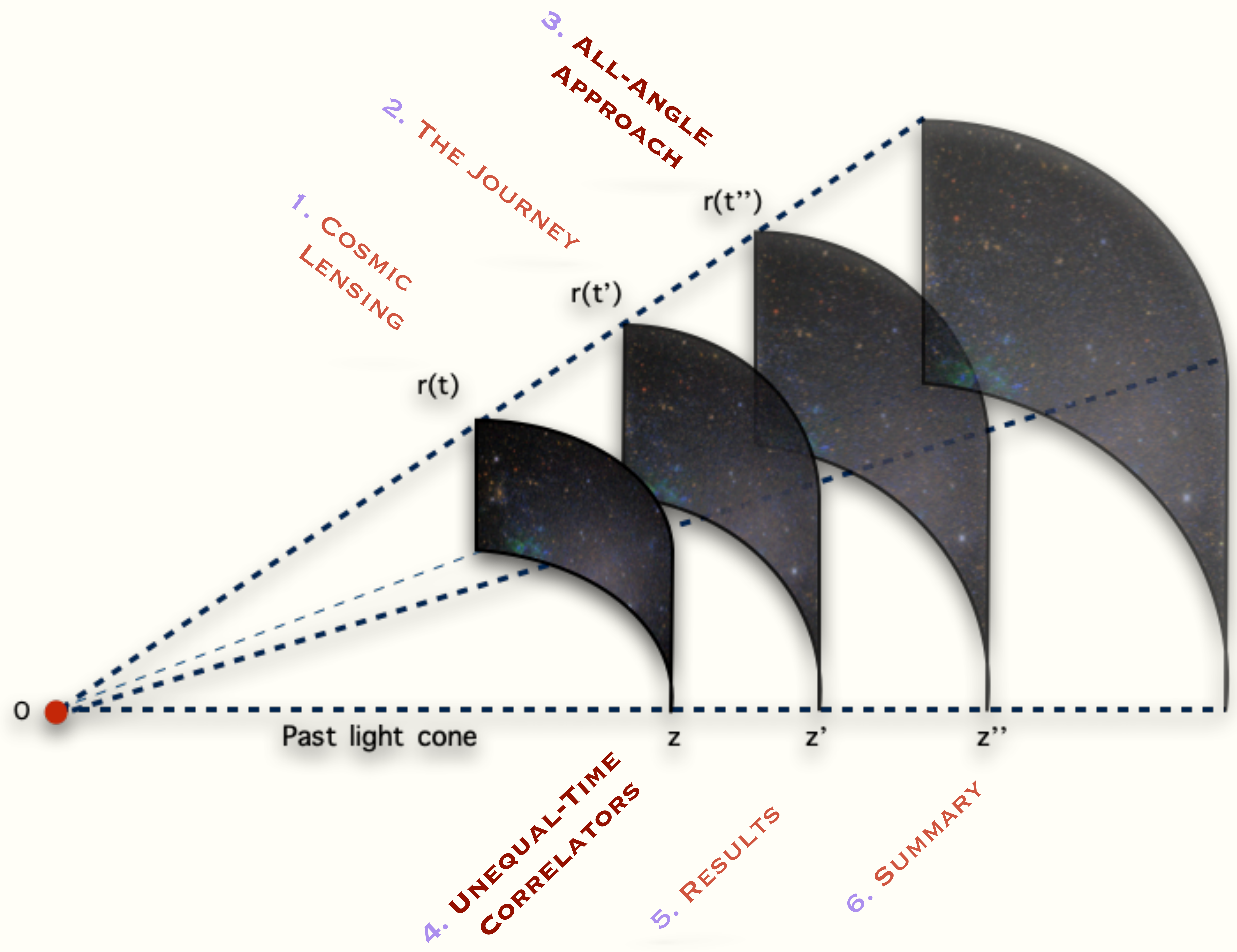
Lucía Fonseca

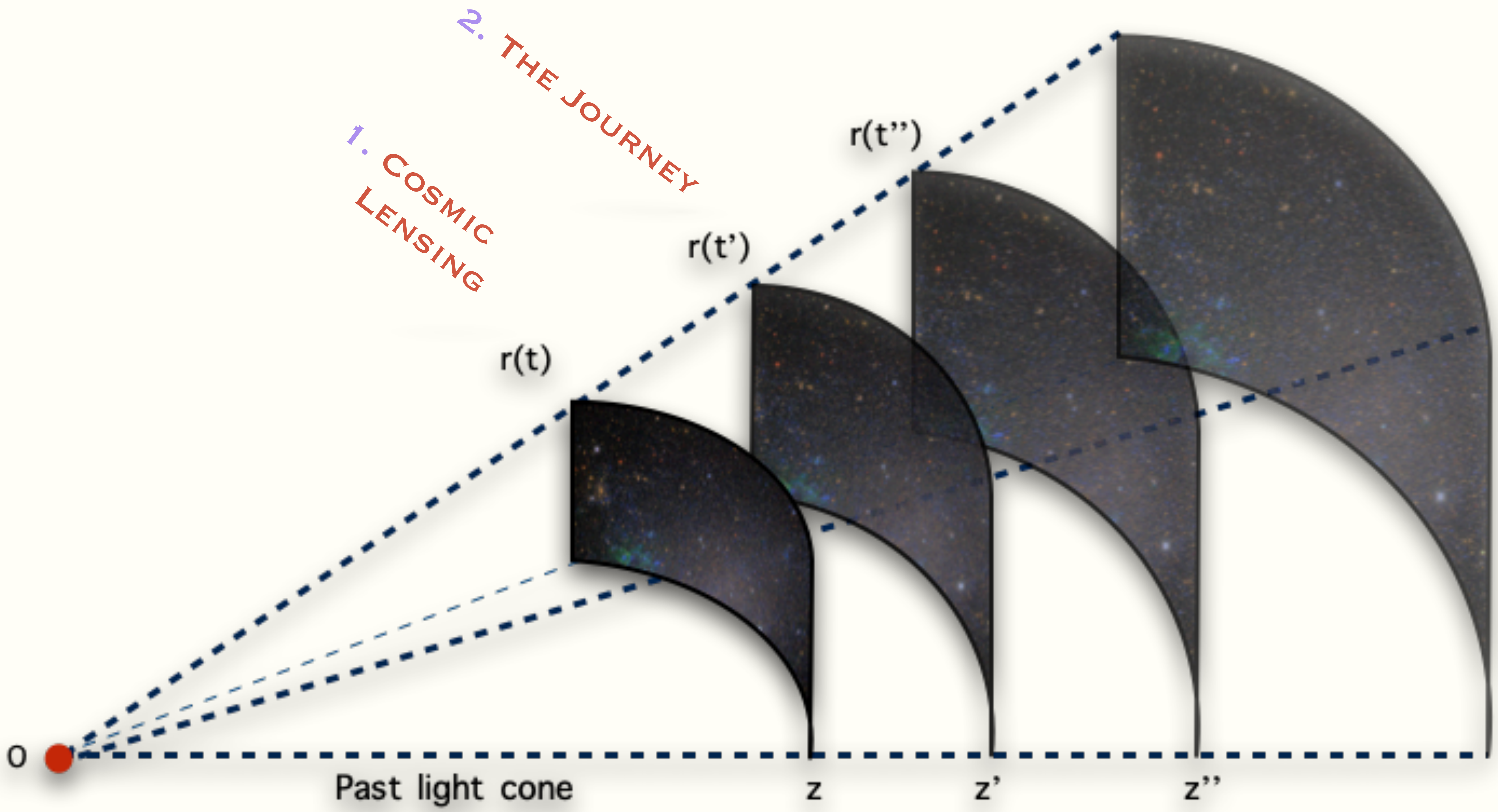
Lucía Fonseca de la Bella

THE JOURNEY OF LIMBER

L.F. de la Bella, N. Tessore and S. Bridle (arXiv 2011.06185)
N. Tessore and **L. F. de la Bella** (in prep)

Python packages UNEQUALPY
CORFU


1. Cosmic Lensing
2. The Journey
3. All-Angle Approach
4. Unequal-Time Correlators
5. Results
6. Summary
r(t)
r(t')
r(t'')
O
Past light cone
z
z'
z''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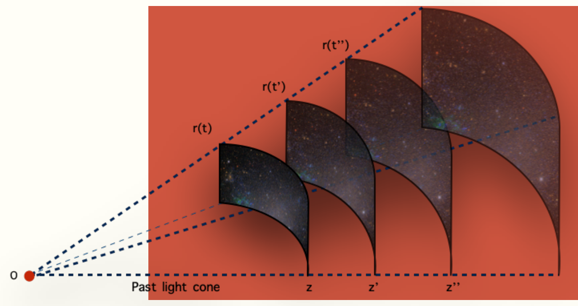
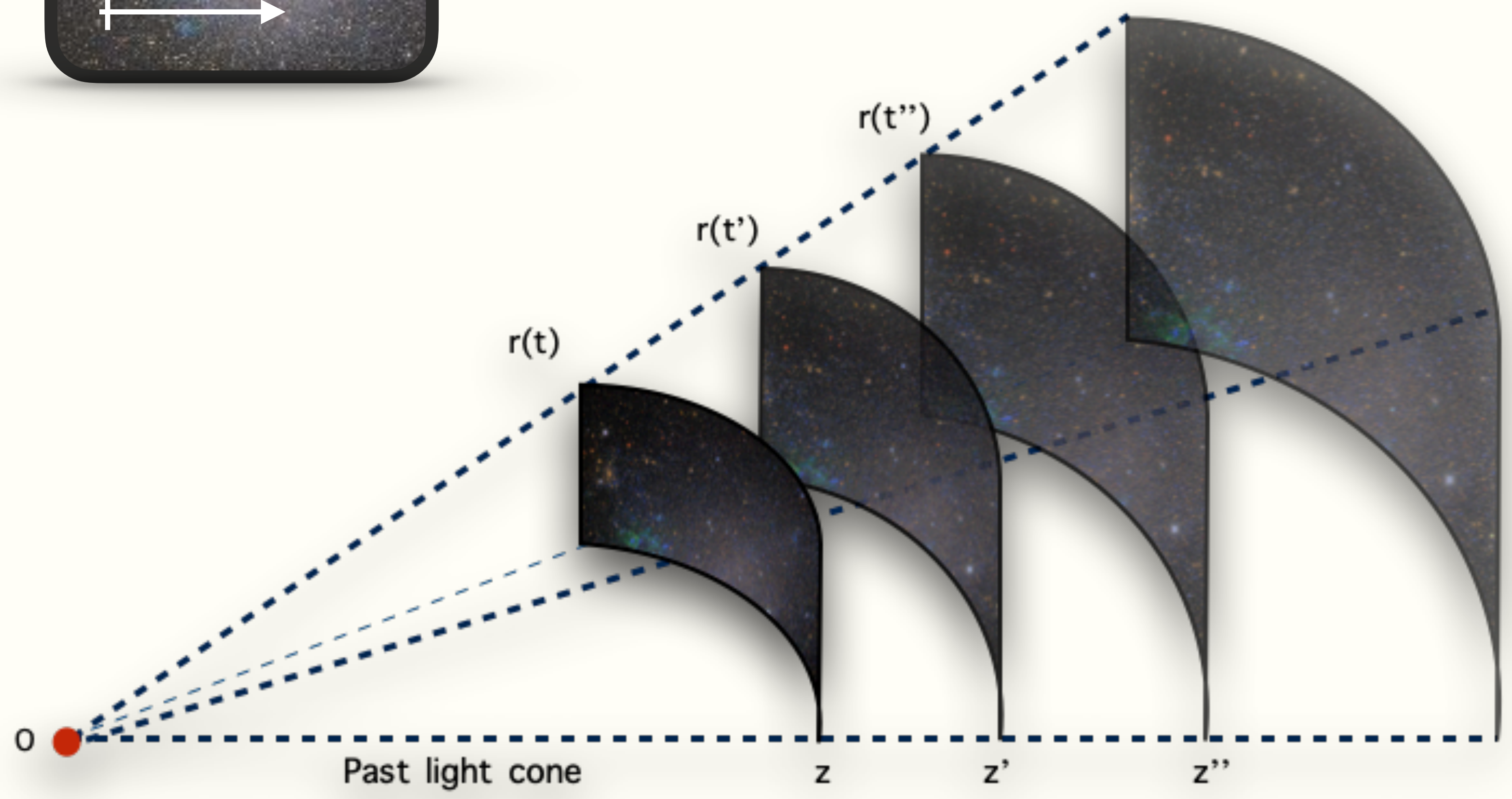


1. Cosmic Lensing



Correlations between fields

- Same time slice: *equal-time correlators*
- Different time slices: *unequal-time correlators*

Examples of fields:

Matter, convergence, cosmic shear

Angular correlation function

$$w(\theta) = \iint_0^\infty dx_1\, dx_2\, f_1(x_1) f_2(x_2)\, \xi(r_{12}; t_1, t_2)$$

FILTERS

UNEQUAL-TIME CORRELATION FUNCTION

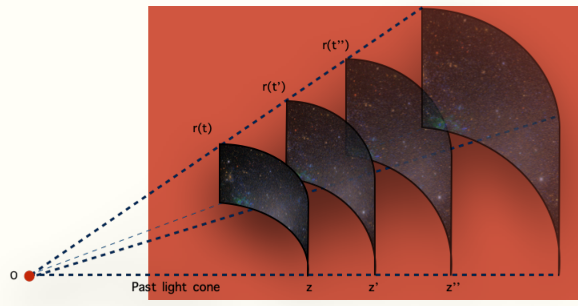
Angular power spectrum

$$C(\ell) = \int_0^\infty \frac{dk}{k^2} \iint_0^\infty dx_1\, dx_2\, f_1(x_1) f_2(x_2)\, j_\ell(kx_1)\, j_\ell(kx_2)\, P(k; t_1, t_2)$$

BESSEL FUNCTIONS

UNEQUAL-TIME POWER SPECTRUM

Note: lookback time = redshift = co-moving distance



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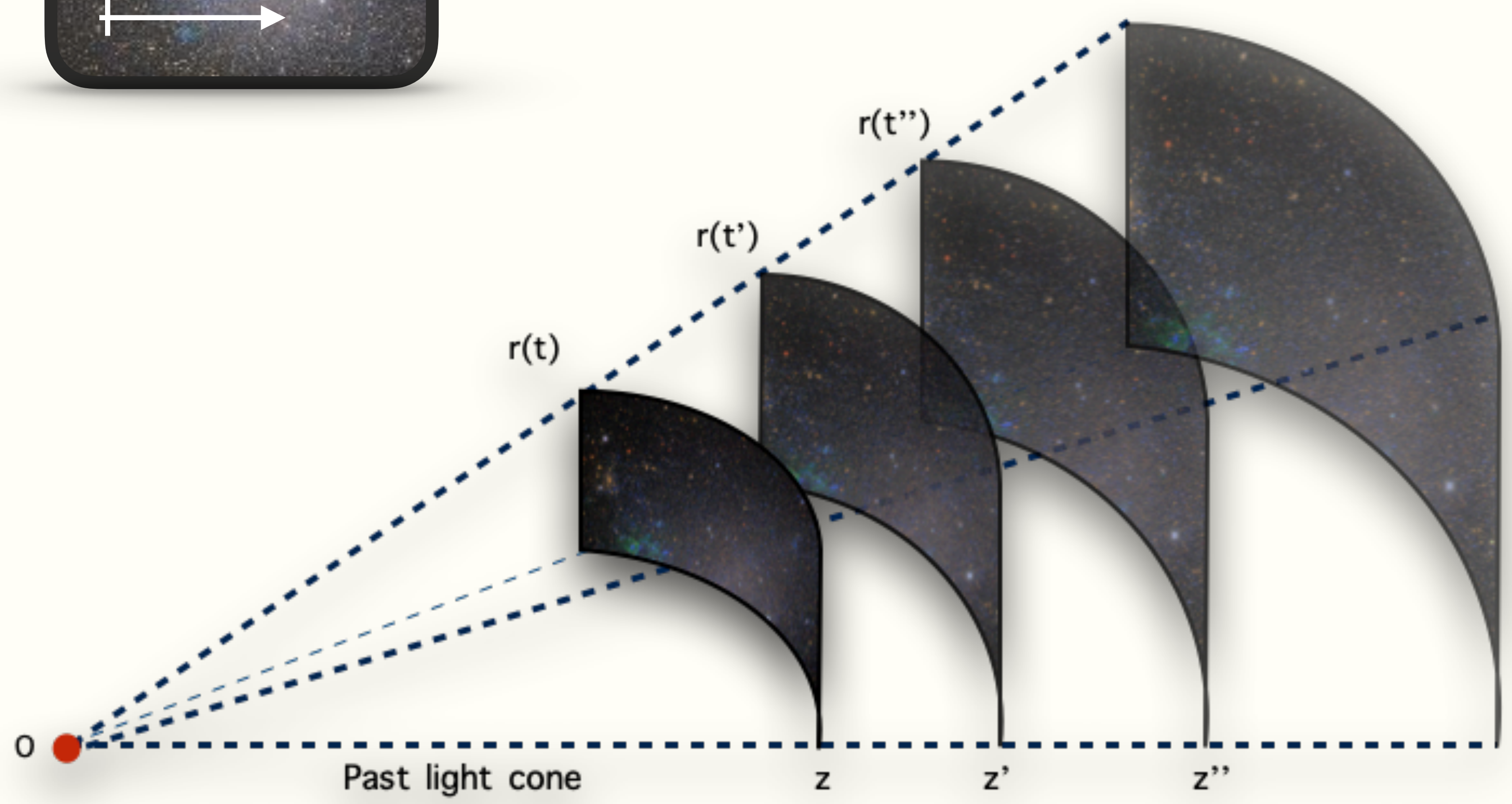

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HARD TO COMPUTE

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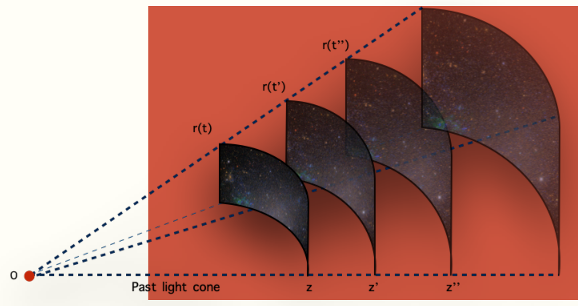
EVEN HARDER TO COMPUTE

BESSEL FUNCTIONS

UNEQUAL-TIME POWER SPECTRUM

This work

Note: lookback time = redshift = co-moving distance



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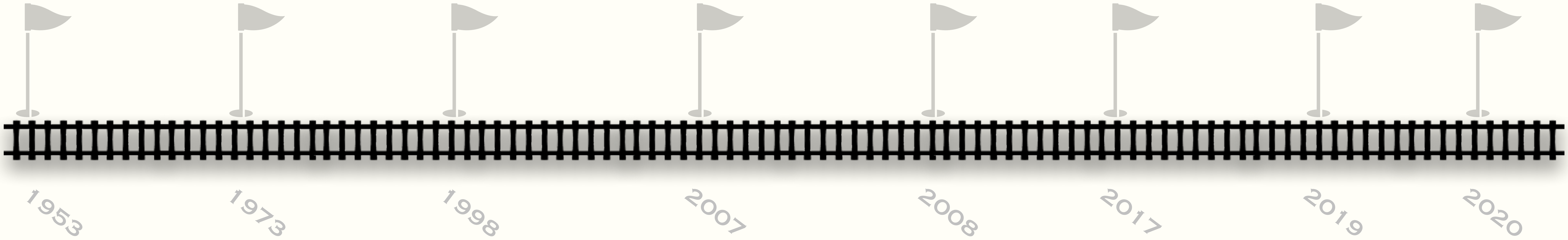
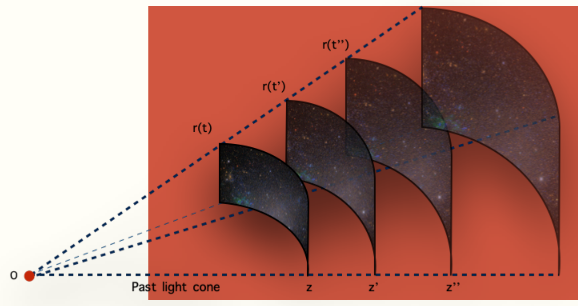
- Filters
- Bessel functions
- Unequal-time correlators





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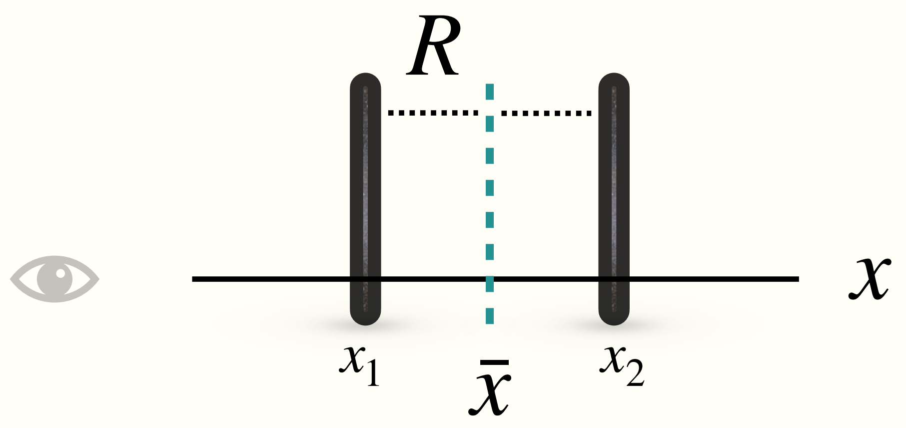
Assumptions:

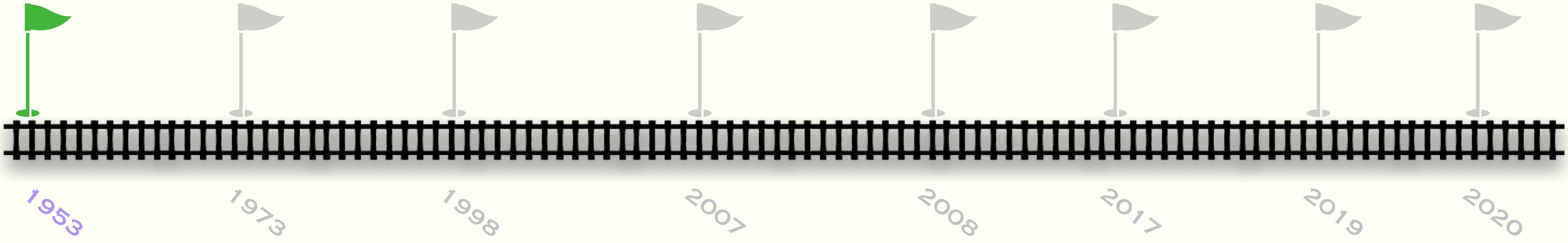
i) Smooth filters
ii) Correlation falls off fast
iii) Small angle separation

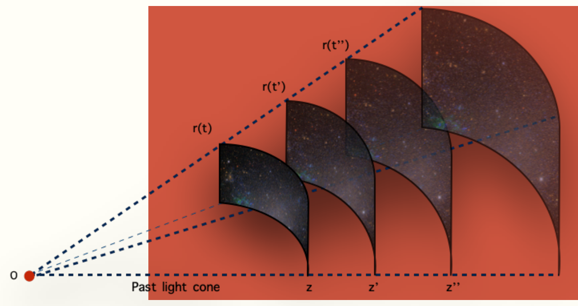


$$w(\theta) \approx \int_0^{\infty} d\bar{x} \int_{-2\bar{x}}^{2\bar{x}} dR\, f_1(\bar{x}) f_2(\bar{x})\, \xi(r_{12}; \bar{t})$$

i) Filters
ii) Mid point

LIMBER





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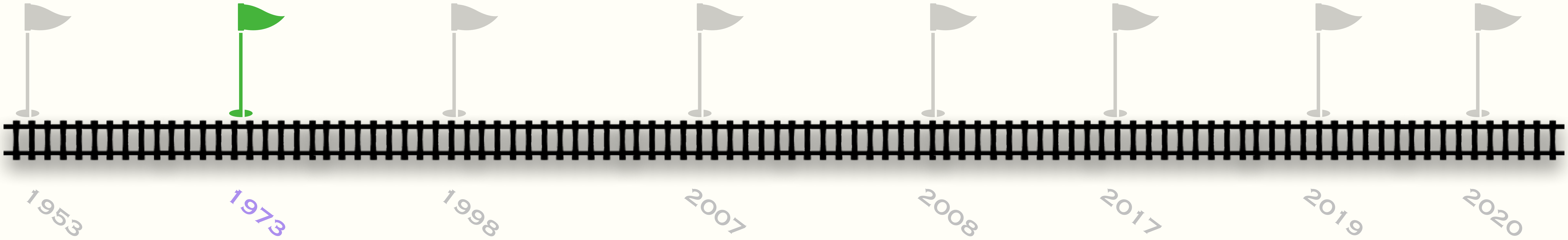
Limber

i) Filters
ii) Mid point

Peebles

i) Sphere
ii) Discrete case



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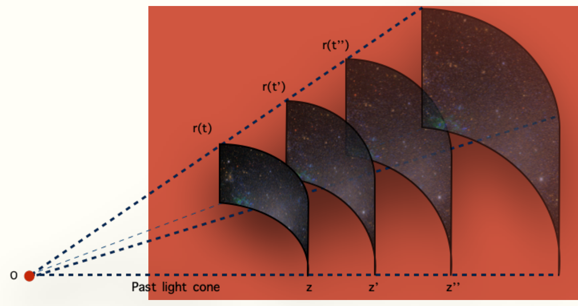

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$$C(\ell) = \int d^2\vec{\theta}\, w(\theta) \mathrm{e}^{-i\vec{\ell}\cdot\vec{\theta}}$$

$$\approx \int dx \frac{f_1(x) f_2(x)}{x^2} P(\ell/x; t)$$

Fourier space
Delta functions

$$w(\theta) \approx \int_0^\infty d\bar{x} \int_{-2\bar{x}}^{2\bar{x}} dR\, f_1(\bar{x})\, f_2(\bar{x})\, \xi(r_{12}; \bar{t}\,)$$

Limber

i) Filters
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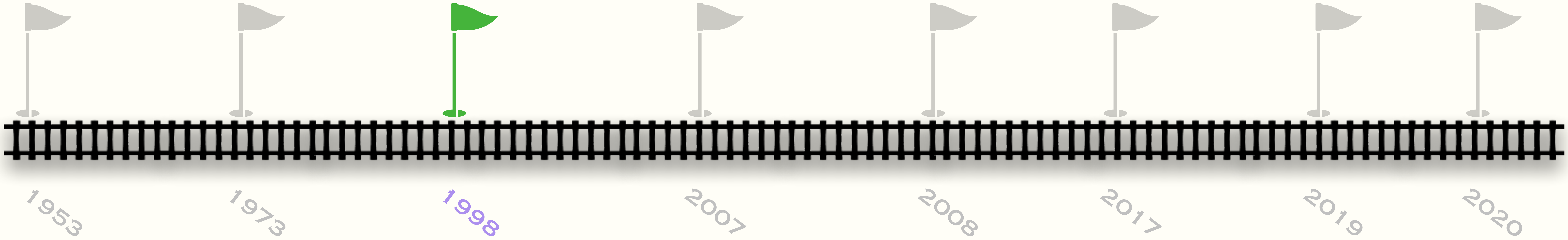
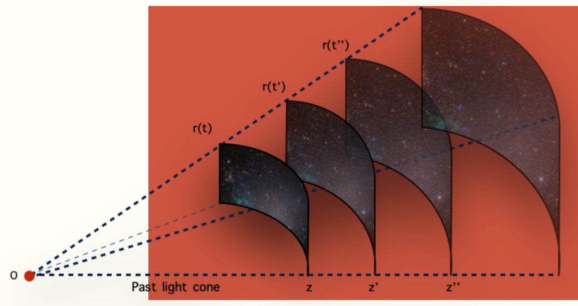
Kaiser

i) Flat sky
ii) Curved universe
iii) Dirac delta





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Accuracy of Limber:

- Small angles: Limber
- Large angles: Thin-layer approximation

- Compares Limber's vs "exact"
- Applies small-angle to "exact"

$$w(\theta) \approx \int_0^\infty d\bar{x} \int_{-2\bar{x}}^{2\bar{x}} dR\, f_1(\bar{x}) f_2(\bar{x})\, \xi(r_{12}; \bar{t}\,)$$

$$C(\ell) \approx \int dx \frac{f_1(x) f_2(x)}{x^2} P(\ell / x; t)$$

Limber (1953)
i) Filters
ii) Mid point

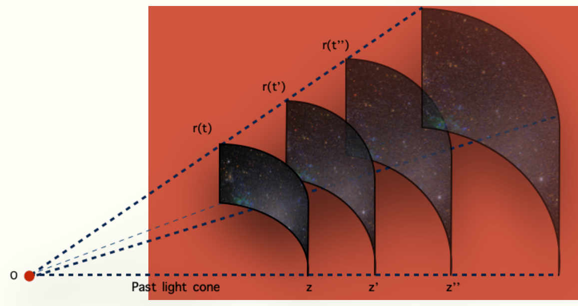
Peebles (1973)
i) Sphere
ii) Discrete case

Kaiser (1998)
i) Flat sky
ii) Curved universe
iii) Dirac delta

Simon (2007)
i) Small and large angles
ii) Thin-layer approx.
iii) **Limber's inaccurate**

2008 — 2017 — 2019 — 2020

2. The Journey



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Dirac delta version:

$$j_\ell(kx) \approx \sqrt{\frac{\pi}{2\nu}}\delta_D(kx - \nu)$$

Series expansion in $1/\nu$

$$\nu = \ell + 1/2$$

$$w(\theta) \approx \int_0^\infty d\bar{x} \int_{-2\bar{x}}^{2\bar{x}} dR\, f_1(\bar{x}) f_2(\bar{x})\, \xi(r_{12}; \bar{t}\,)$$

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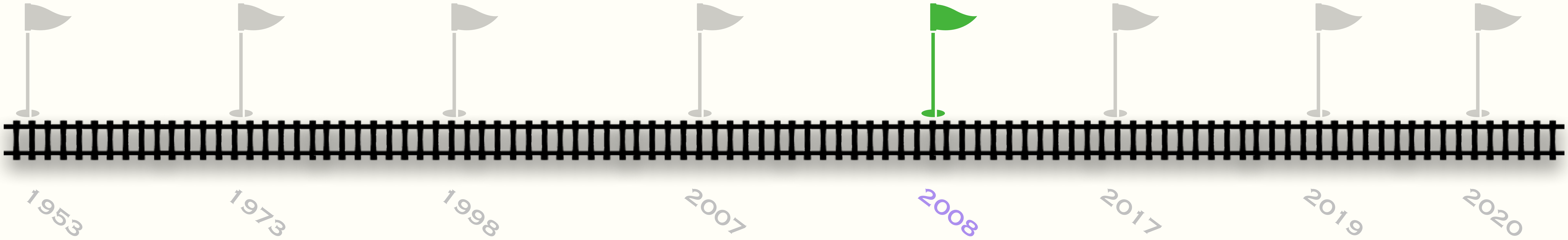
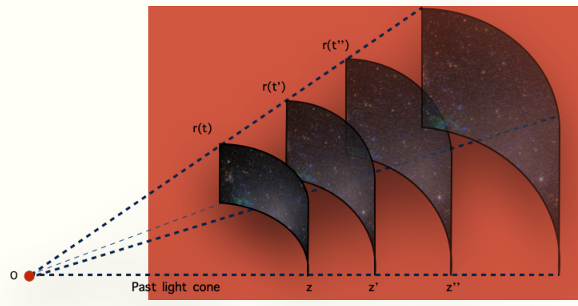
LoVerde et al (2008)
- i) Post-Limber
- ii) Divergence small ℓ





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Geometric approximation (large scales)

$$P(k; t_1, t_2) \approx \sqrt{P(k, t_1) P(k, t_2)}$$

$$j_\ell(kx) \approx \sqrt{\frac{\pi}{2\nu}}\, \delta_D(kx - \nu)$$

$$C(\ell) \approx \frac{\pi}{2\nu} \int \frac{dk}{k^2} f_1(\nu/k) f_2(\nu/k) P(k, \nu/k)$$

$$w(\theta) \approx \int_0^\infty d\bar{x} \int_{-2\bar{x}}^{2\bar{x}} dR\; f_1(\bar{x}) f_2(\bar{x})\, \xi(r_{12}; \bar{t}\,)$$

$$C(\ell) \approx \int dx \frac{f_1(x) f_2(x)}{x^2} P(\ell/x; t)$$

Expansion $1/\nu$

Limber
i) Filters
ii) Mid point

Peebles
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i) Small and large angles
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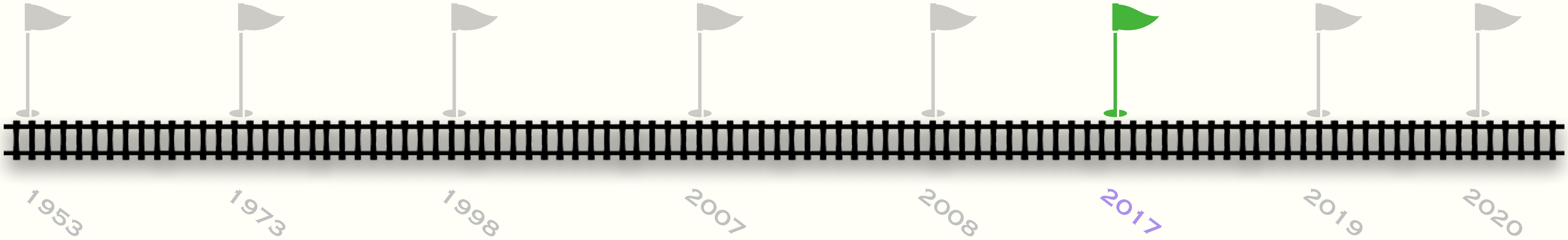
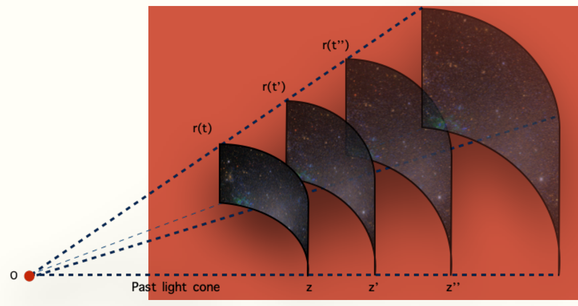
LoVerde et al
i) Post-Limber
ii) **Divergence small ℓ**

Lemos et al
i) Geometric
ii) Dirac delta
iii) **Valid large ℓ**
iv) **Valid small k**





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Linear — Non-linear

$$C(\ell) \approx \int_0^\infty \frac{dk}{k^2}(\ldots)\, P_{lin}(k; t_1, t_2) + \int_0^\infty \frac{dk}{k^2}(\ldots)\, P_{Nl}(k; t_1, t_2)$$

Exact

$$P(k; t_1, t_2) \equiv \sqrt{P(k, t_1) P(k, t_2)}$$

No unequal-time
- Dirac delta $\ell >> 1$
- Geometric $k << 1$?
- NL Physics?

Limber (1953)

$$w(\theta) \approx \int_0^\infty d\bar{x} \int_{-2\bar{x}}^{2\bar{x}} dR\, f_1(\bar{x}) f_2(\bar{x})\, \xi(r_{12}; \bar{t})$$

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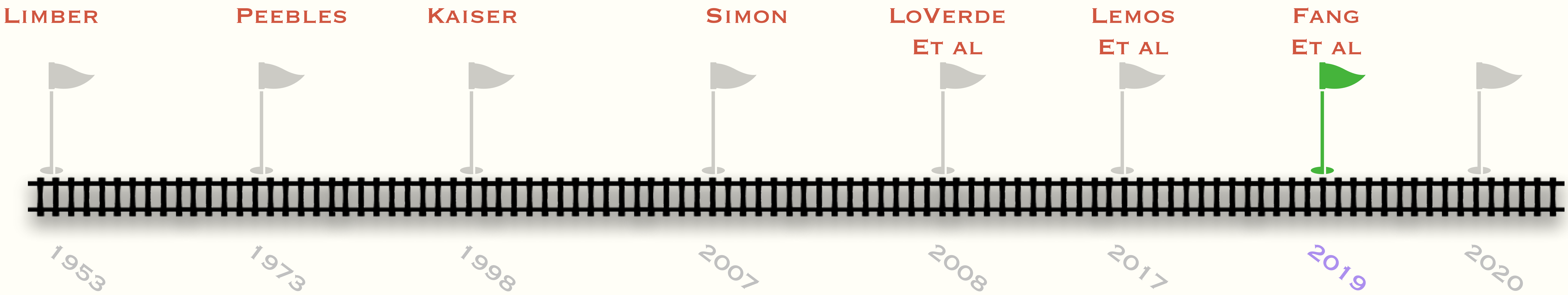
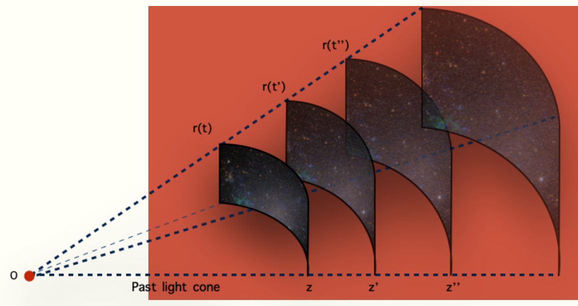
Fang et al (2019)

i) FFTLog
ii) Split regimes
iii) **Limber Inaccurate**
iv) **Accuracy large k?**





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Issues

- Limber's accuracy
- Divergence on large angles
- Geometric app good on large scales
- Accuracy of non-linear physics

Need

- ☆ Unequal-time correlators
- ☆ All-angle calculations

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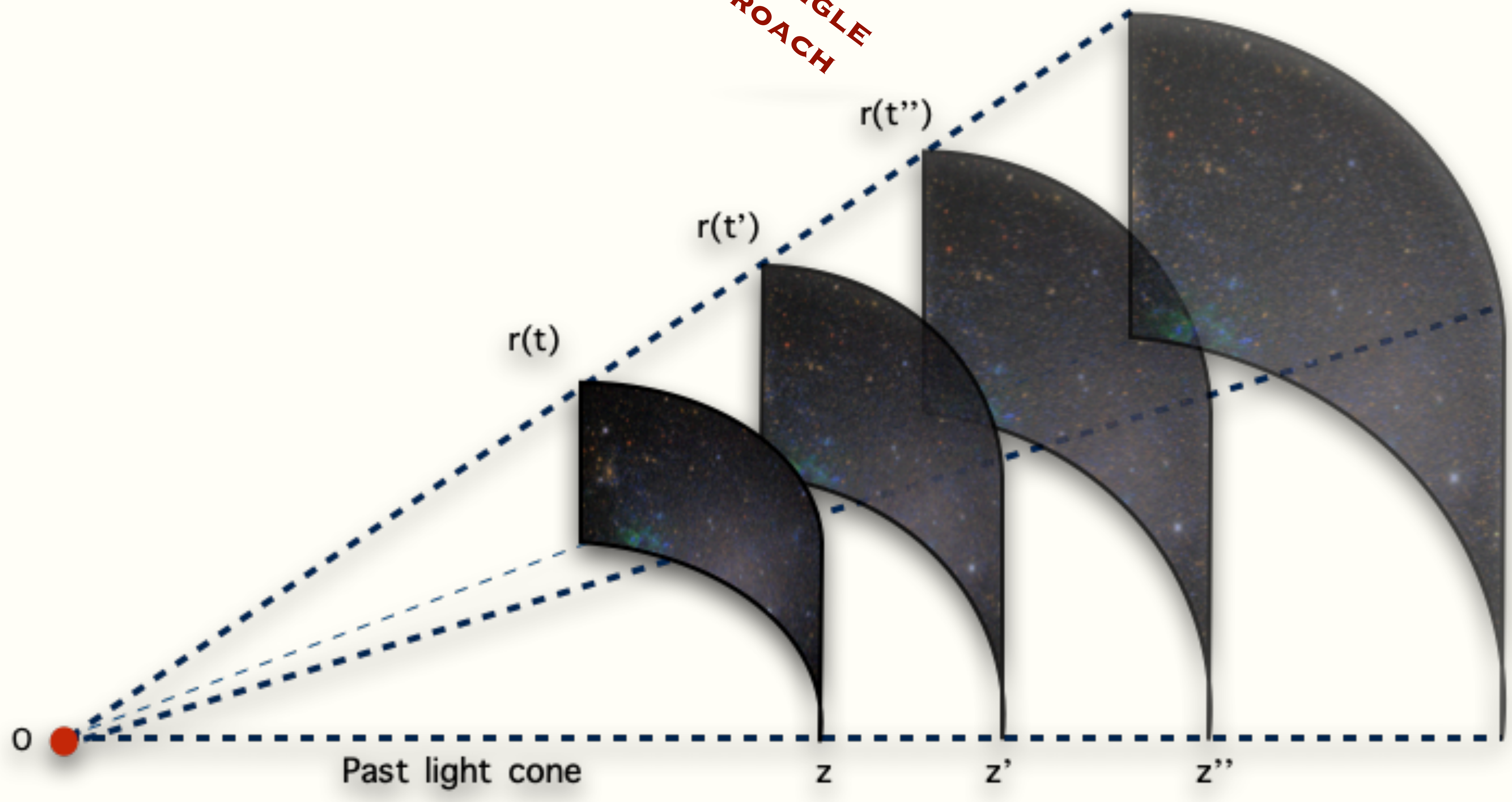
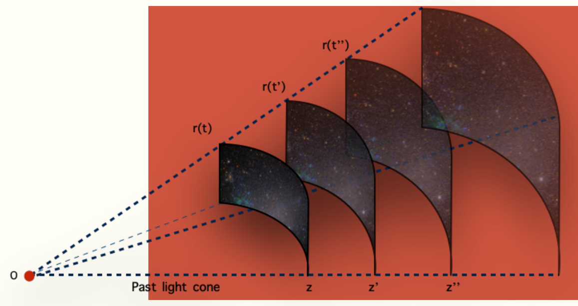
This work (2020)

N. Tessore, L. F. de la Bella (in prep)
Python package **CORFU**

3. **All-Angle Approach**





3. All-angle approach

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- Need accurate unequal-time power spectrum
- Deal with non-linear physics (one-loop, EFT)
- Impact on weak lensing. How?
  - Midpoint approximation ★
  - Compare with geometric approx.
- Analyse regimes of validity
- Drop approximations
- Python package UNEQUALPY

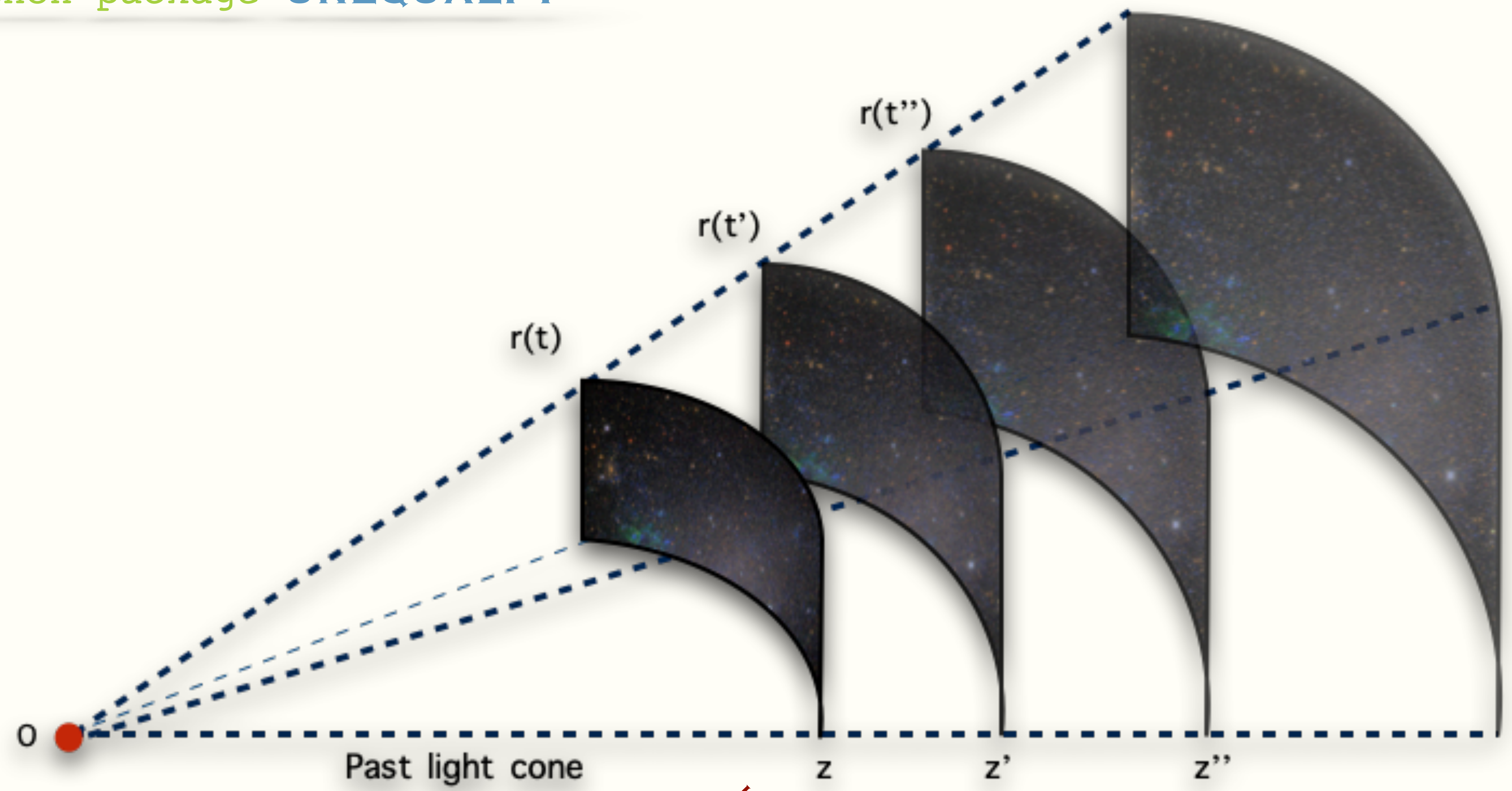
- Need exact calculation at all angular scales
- Python package CORFU
  - FFTLog methods
  - Inverse Fourier transform

$$P(k; t_1, t_2) \longrightarrow \xi(r; t_1, t_2)$$

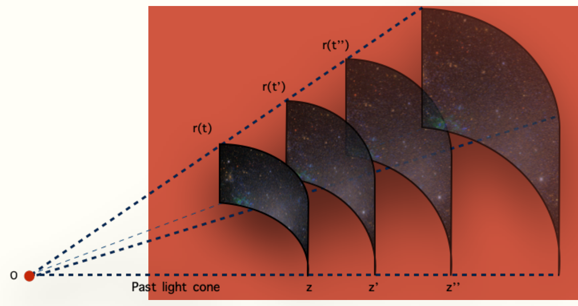
  - Legendre polynomials

$$w(\theta) \longrightarrow C(\ell)$$

L. F. de la Bella, N. Tessore and S. L. Bridle (arXiv 2011.06185)
Python package **UNEQUALPY**



4. UNEQUAL-TIME CORRELATORS



4. Unequal-time Correlators

2point functions

Homogeneous
Isotropic

- EQUAL-TIME POWER SPECTRUM

  Same time slice

$$(2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2)P(k,z) := \langle \delta(\mathbf{k}_1,z)\delta^*(\mathbf{k}_2,z)\rangle$$

- UNEQUAL-TIME POWER SPECTRUM

  Different time slices

$$(2\pi)^3\delta_D(\mathbf{k}_1+\mathbf{k}_2)P(k,z_1,z_2) := \langle \delta(\mathbf{k}_1,z_1)\delta^*(\mathbf{k}_2,z_2)\rangle$$

LINEAR THEORY

$$P(k,z) = D(z)^2 P_{11}(k)$$

$$P(k,z_1,z_2) = D(z_1)D(z_2)P_{11}(k)$$

@ $z_1 = z_2$

$$P(k,z,z) = P(k,z)$$

Non-linear

ONE-LOOP THEORY

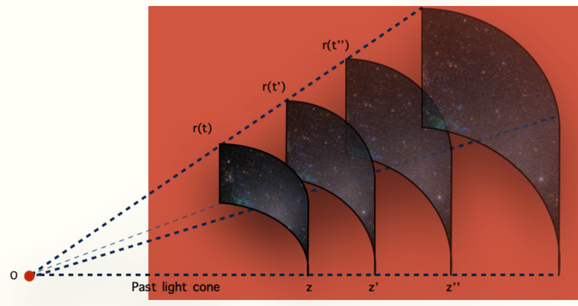
$$P(k,z_1,z_2) = P_{11}(k,z_1,z_2) + P_{22}(k,z_1,z_2) + P_{13}(k,z_1,z_2)$$

i) STANDARD PERTURBATION THEORY
ii) EFFECTIVE FIELD THEORY

NL physics → counterterms

$$P_{EFT} = P_{SPT} - c^2k^2P_{11}$$

- This work shows unequal-time EFT breaks
- New idea: **the midpoint approximation!**

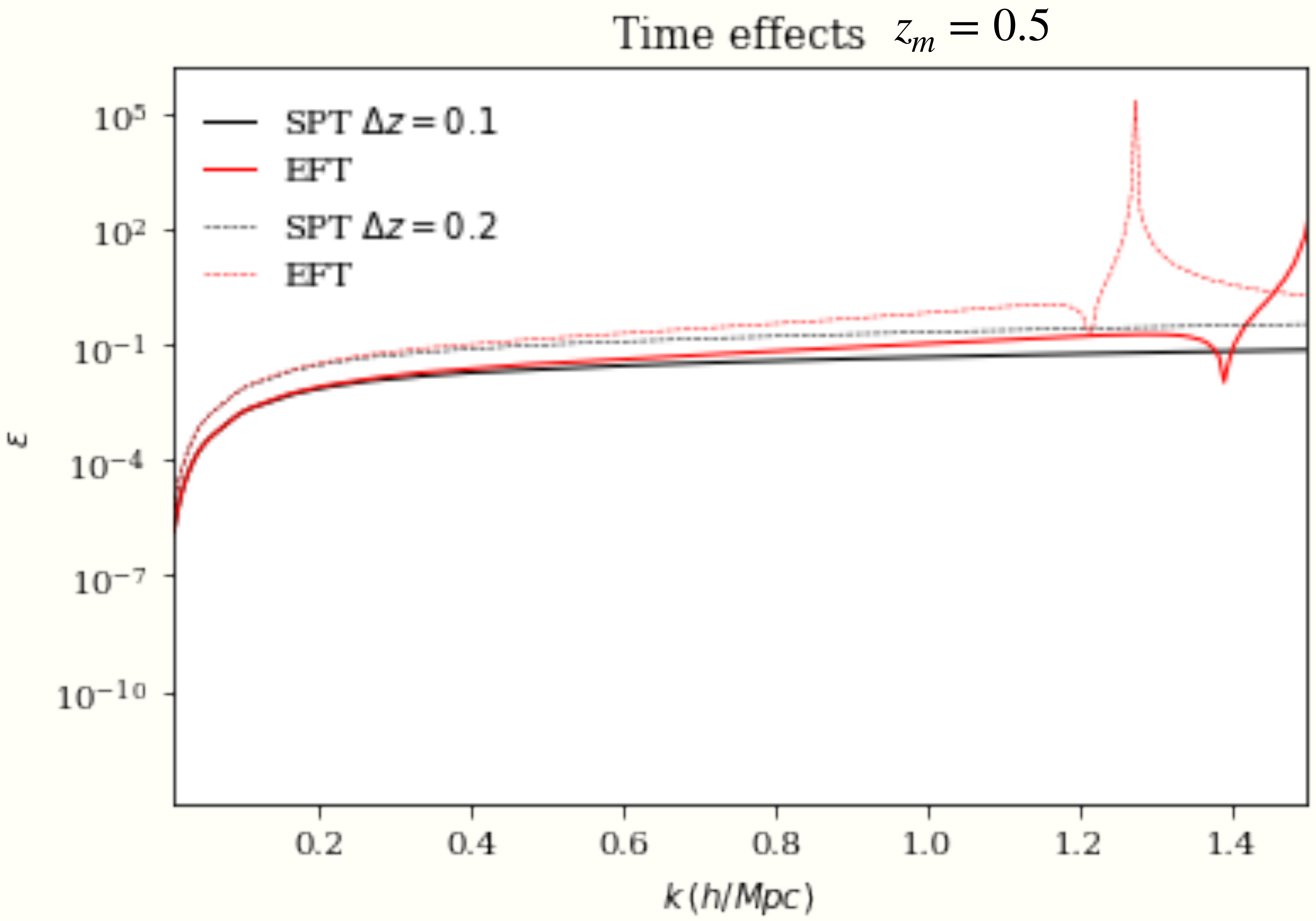


4.1. Time and non-linear effects

Time

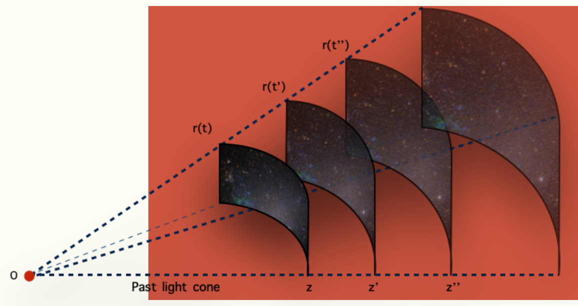
SPT Geometric*	**EFT Geometric***
SPT Unequal-time	**EFT Unequal-time**

Non-linear



$$* \quad P(k; t_1, t_2) \approx \sqrt{P(k, t_1)P(k, t_2)}$$

$$\frac{\Delta P}{P^2} = \frac{|P_{theory}(k, z_1)P_{theory}(k, z_2) - P_{theory}(k; z_1, z_2)^2|}{P_{theory}(k; z_1, z_2)^2}$$

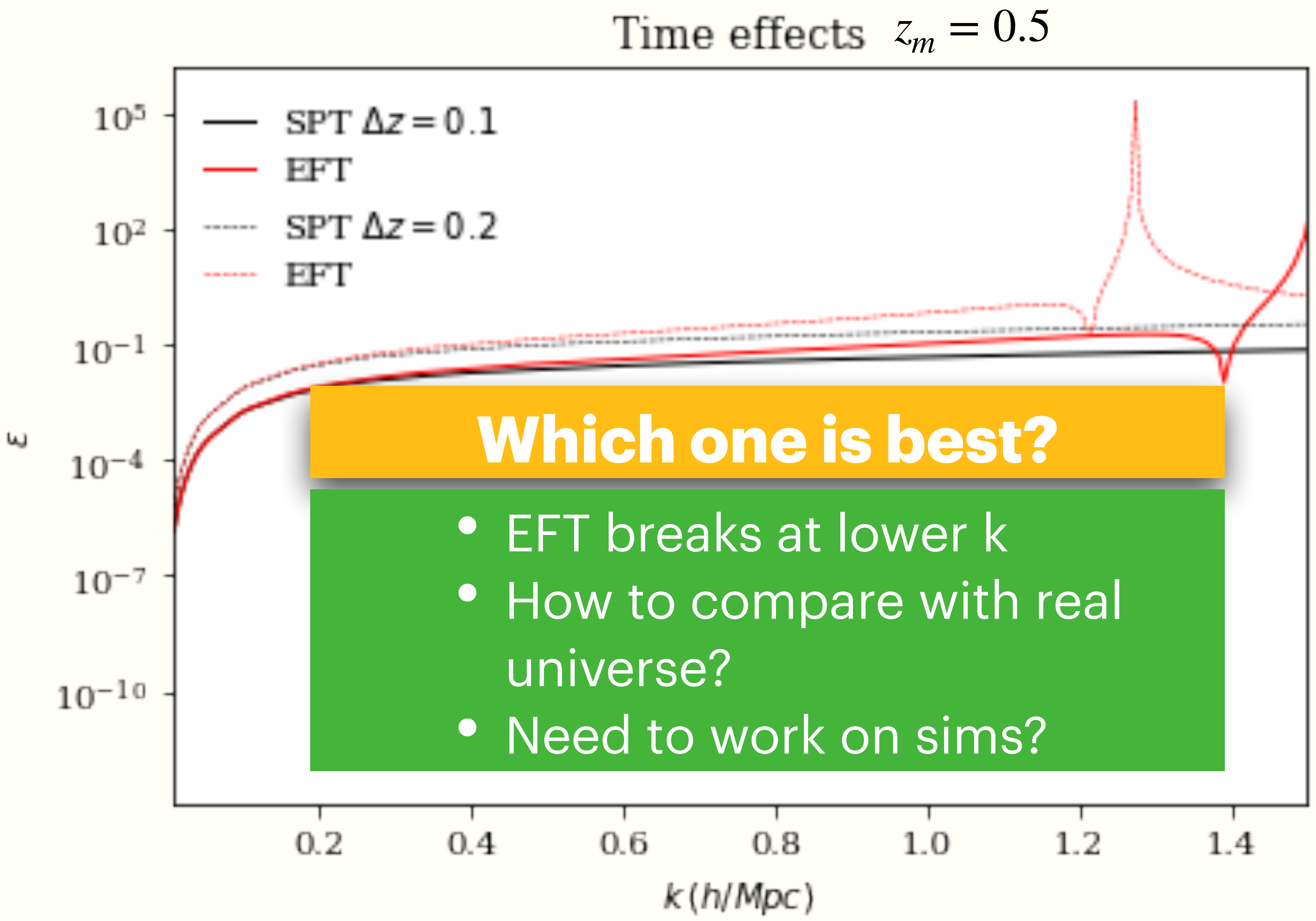


4.1. Time and non-linear effects

Time ↓

	SPT Geometric*	**EFT Geometric***
	SPT Unequal-time	**EFT Unequal-time**

Non-linear →

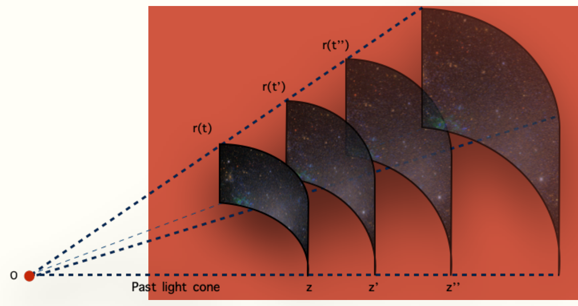


Which one is best?

- EFT breaks at lower k
- How to compare with real universe?
- Need to work on sims?

* $P(k; t_1, t_2) \approx \sqrt{P(k, t_1)P(k, t_2)}$

$$\frac{\Delta P}{P^2} = \frac{|P_{theory}(k, z_1)P_{theory}(k, z_2) - P_{theory}(k; z_1, z_2)^2|}{P_{theory}(k; z_1, z_2)^2}$$



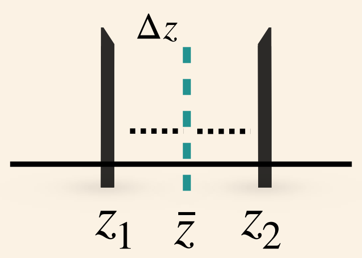
4.2. Midpoint approximation

- Unequal-time power spectrum

$$P(k; z_1, z_2)$$

- Geometric approximation

$$\sqrt{P(k, z_1)P(k, z_2)}$$



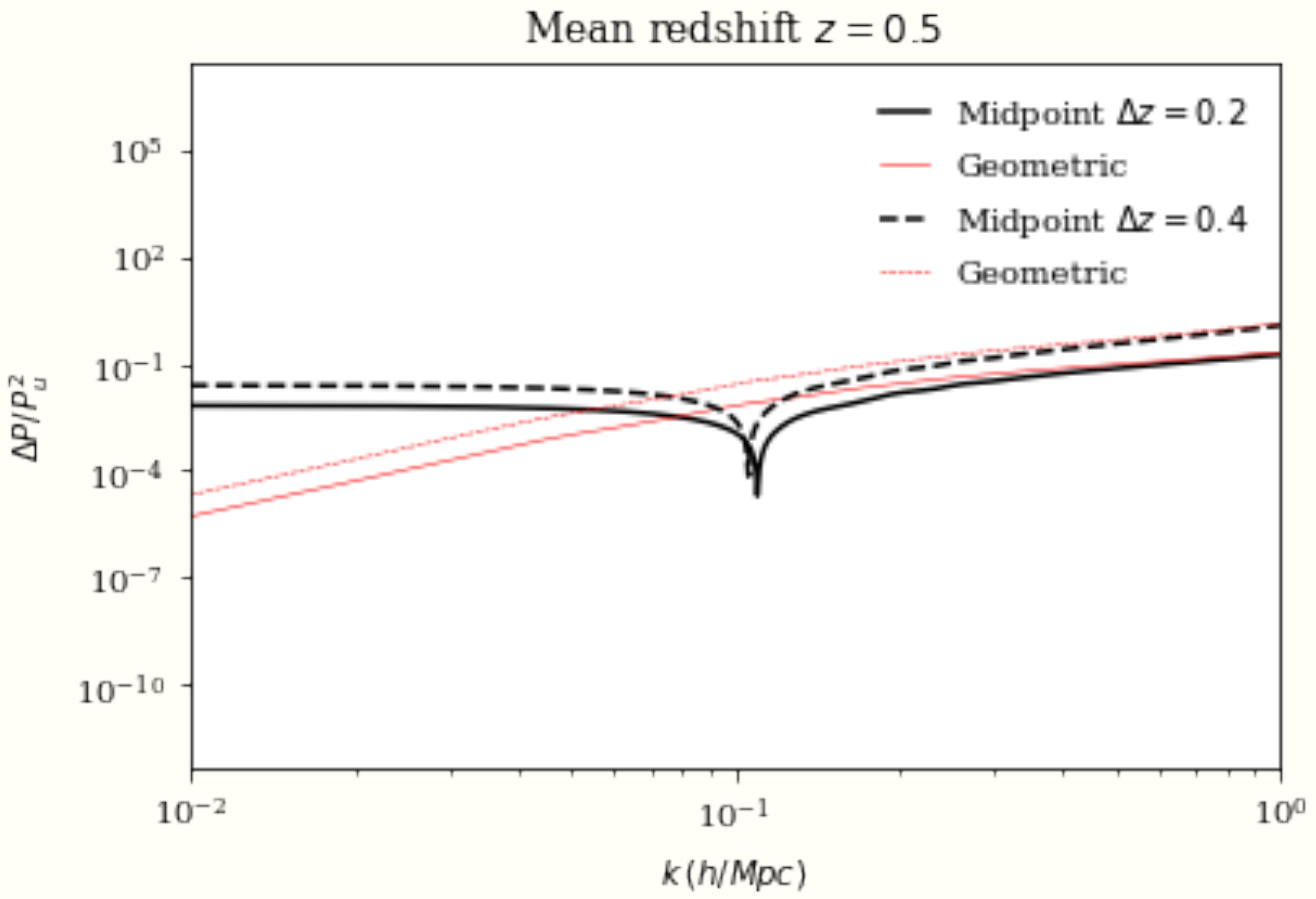
- Midpoint approximation

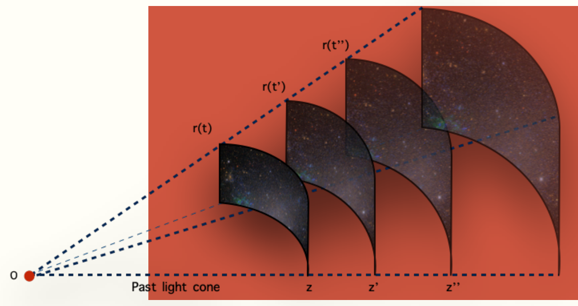
★ $P(k; z_m)$

- More accurate for small separation
- Geometric approximation
  - Good on very large scales
  - Error increases on smaller scales

- Midpoint approximation
  - Not perfect on large scales
  - Error much more stable
  - Better prediction on small scales

- EFT does not improve SPT





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- Geometric approximation

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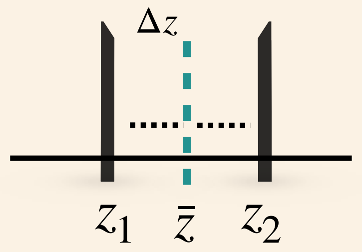

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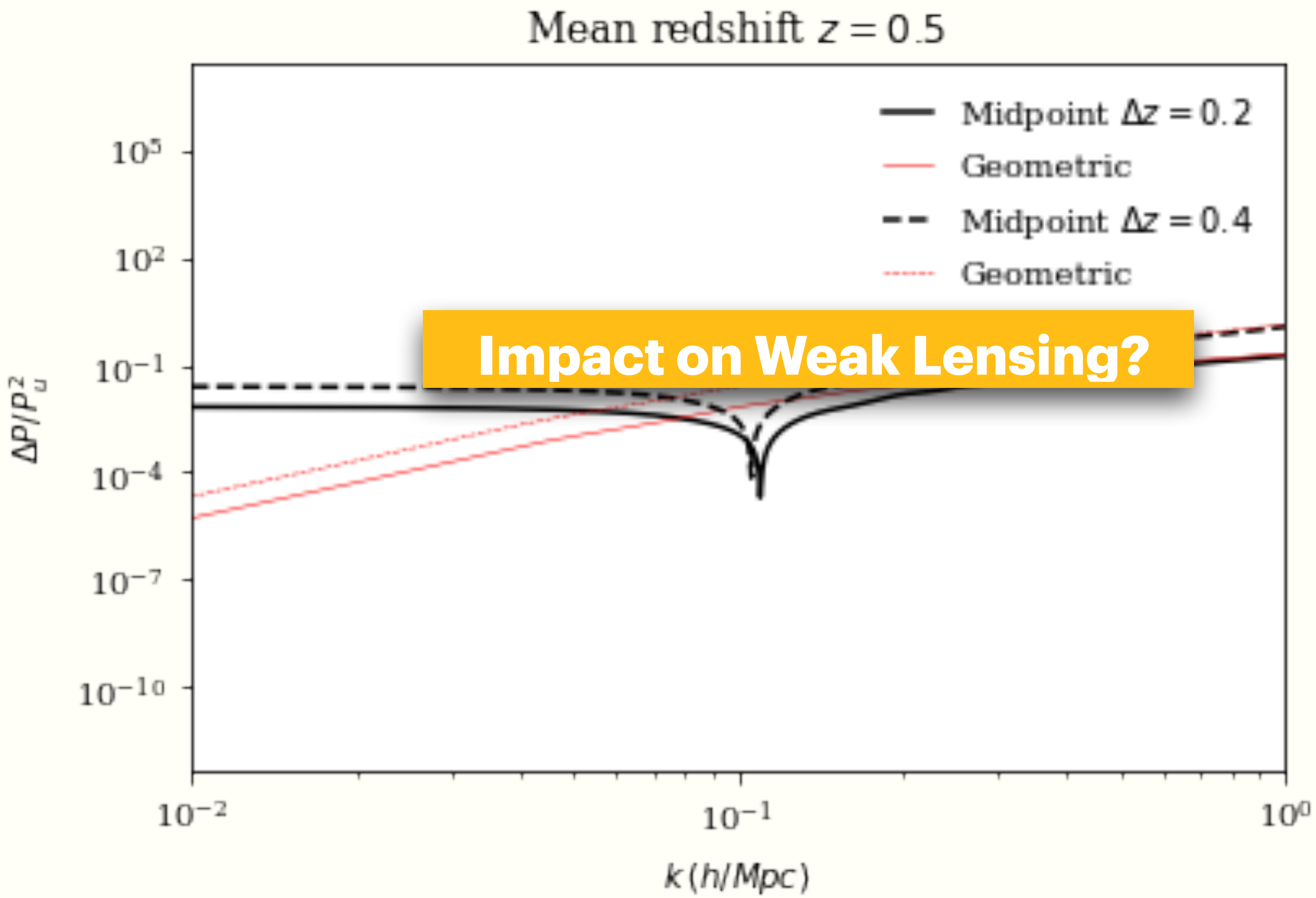
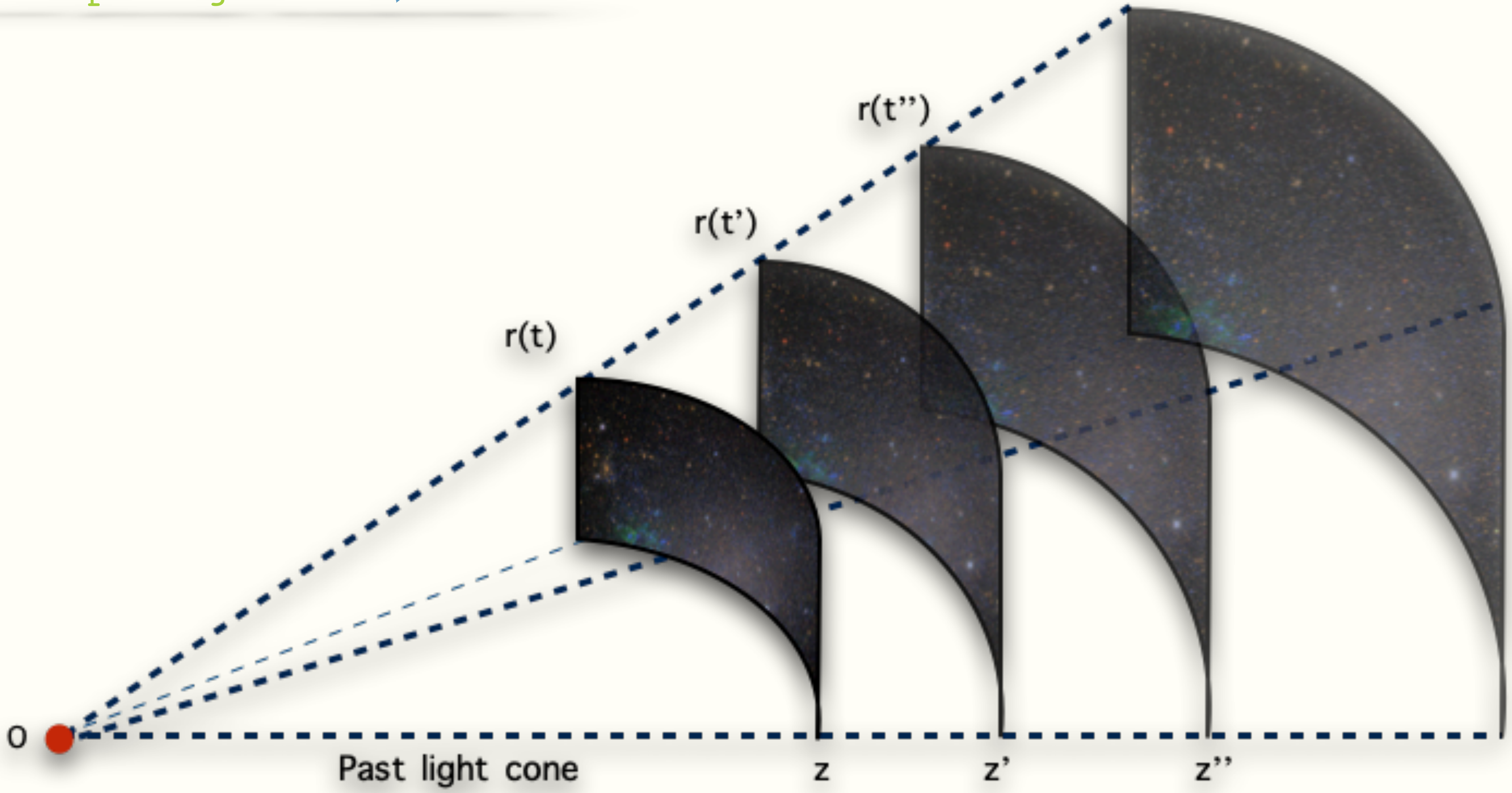
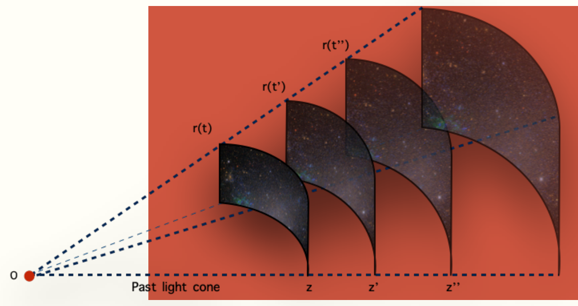


L. F. de la Bella, N. Tessore and
S. L. Bridle (arXiv 2011.06185)
Python package **UNEQUALPY**





5. Results

Exact

$$C_{ab}^{(i,j)}(\ell) = \frac{2}{\pi}\int_0^\infty dk k^2 \iint_0^\infty dx_1\, dx_2\;\, f_a^i(x_1) f_b^j(x_2)\, j_\ell(kx_1)\, j_\ell(kx_2)\, P(k; t_1, t_2)$$

CORFU

$P(k; t_1, t_2) \longrightarrow \xi(r; t_1, t_2)$

$w(\theta) \longrightarrow C(\ell)$

Limber

$$C_{ab}^{(i,j)}(\ell) \approx \frac{1}{\nu}\int_0^\infty dk k^2 f_a^i(\nu/k) f_b^j(\nu/k) P(k; \nu/k)$$

Geometric

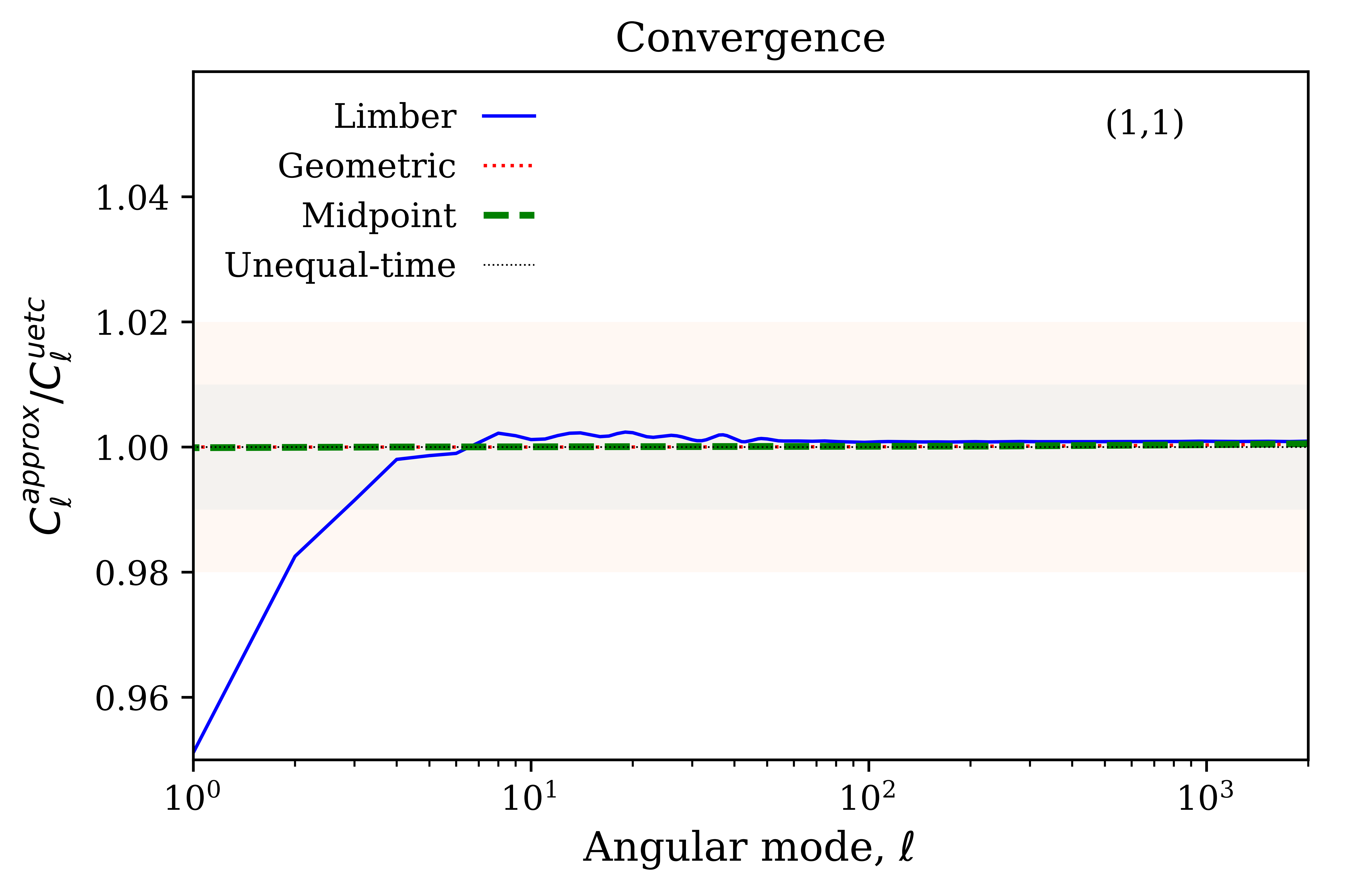
$$C_{ab}^{(i,j)}(\ell) \approx \frac{2}{\pi}\int_0^\infty dk k^2 \int_0^\infty dx_1\, f_a^i(x_1) j_\ell(kx_1)\sqrt{P(k; x_1)} \int_0^\infty dx_2 f_b^j(x_2) j_\ell(kx_2)\sqrt{P(k; x_2)}$$

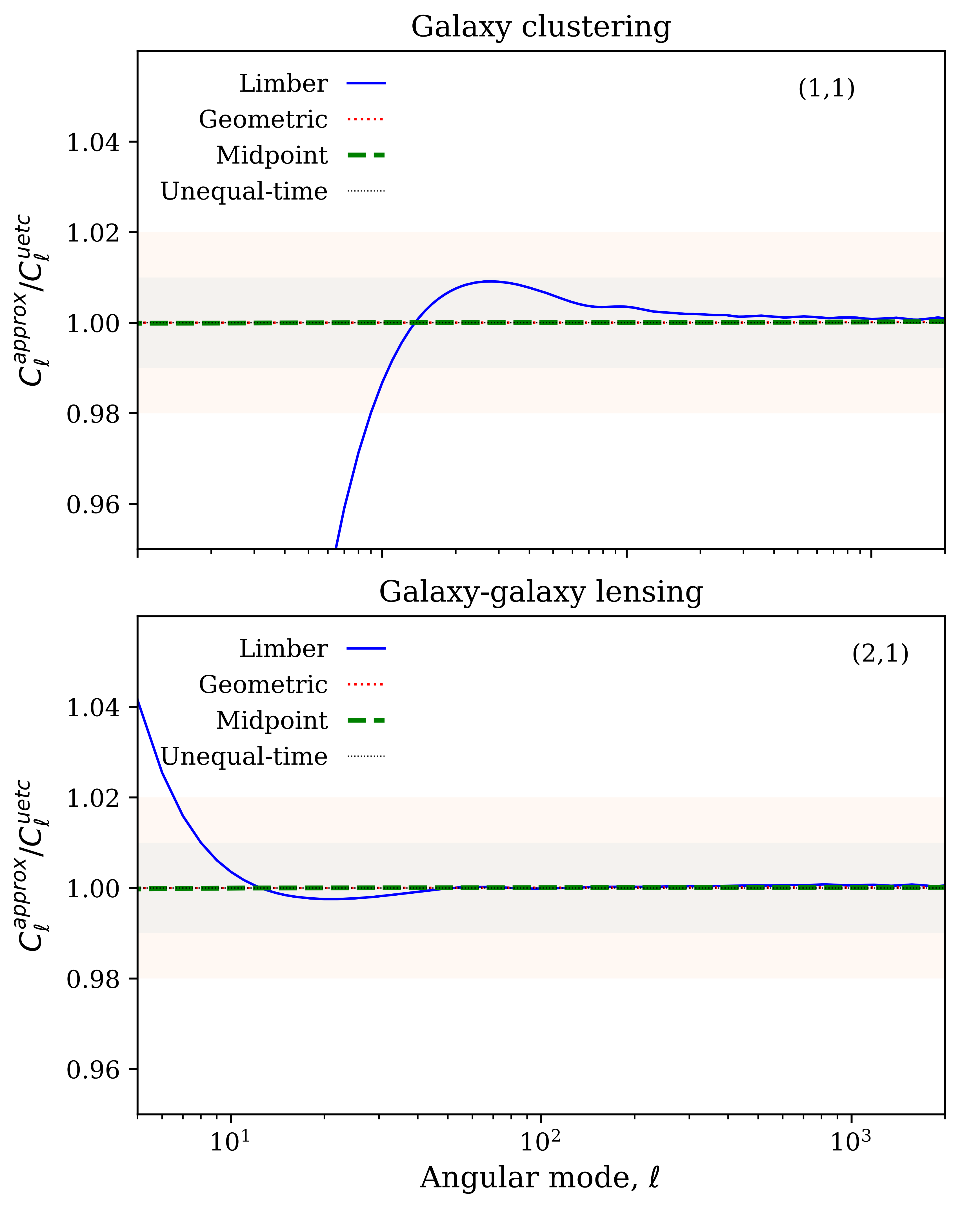
Midpoint

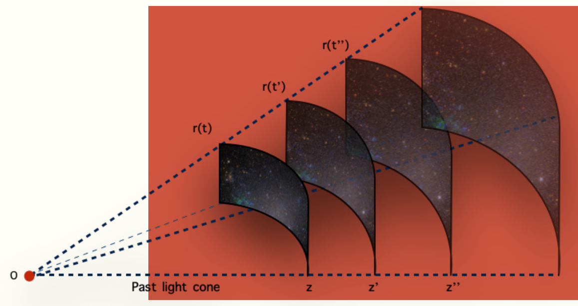
$$C_{ab}^{(i,j)}(\ell) \approx \frac{2}{\pi}\int_0^\infty dk k^2 \iint_0^\infty dx_1\, dx_2\;\, f_a^i(x_1) f_b^j(x_2)\, j_\ell(kx_1)\, j_\ell(kx_2)\, P\left(k; \frac{x_1 + x_2}{2}\right)$$

5. Results

DES-Y1 DATA

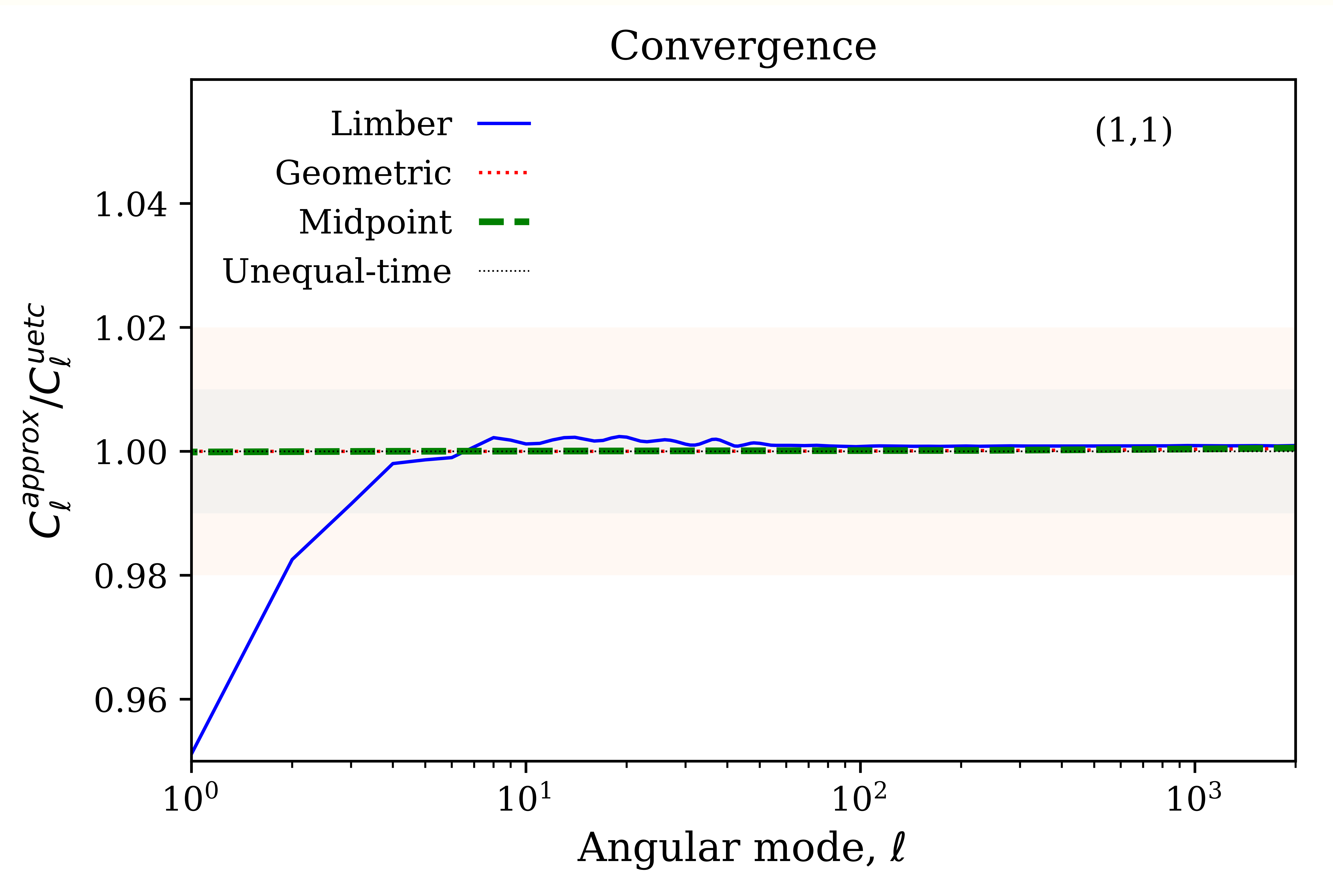


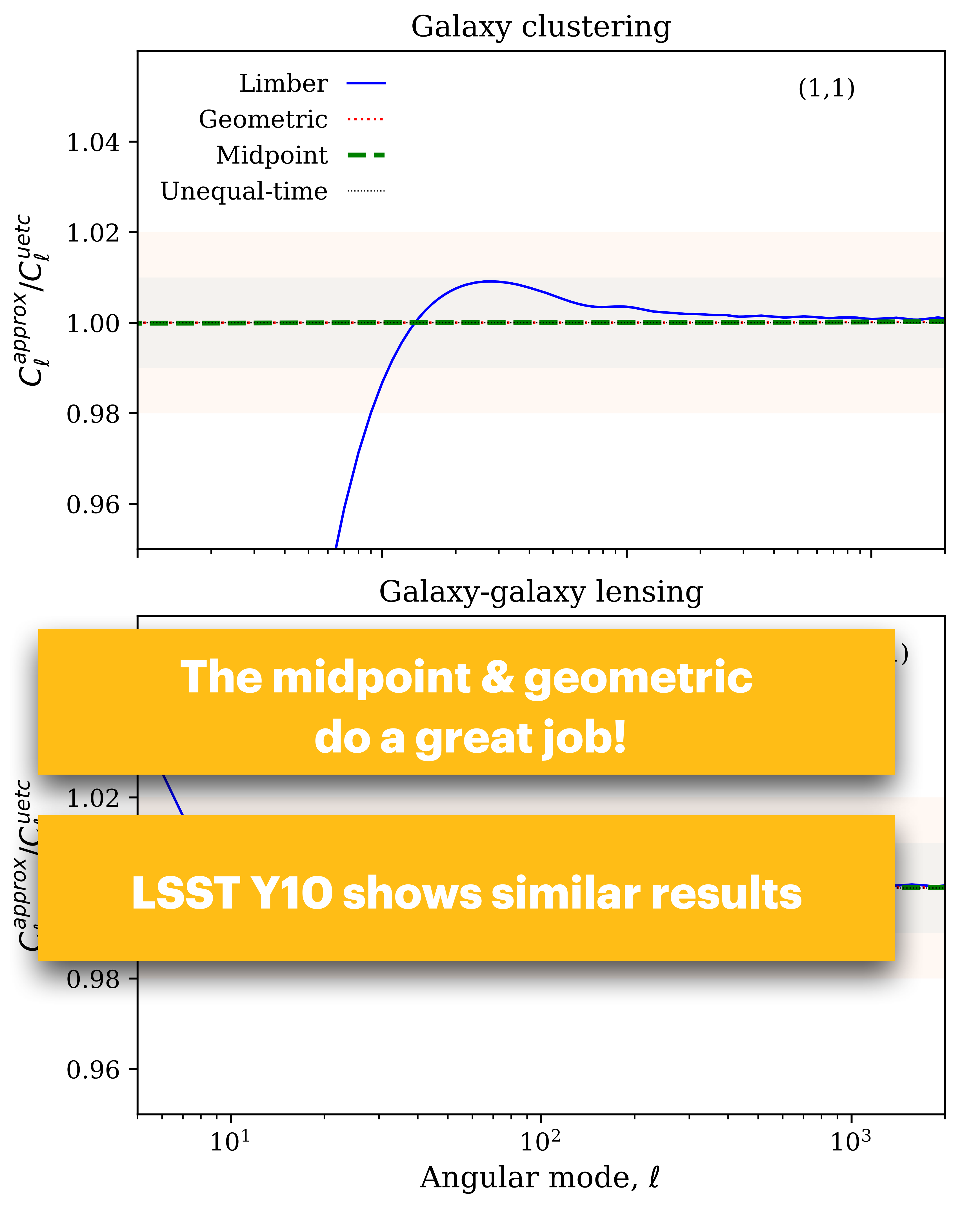




5. Results

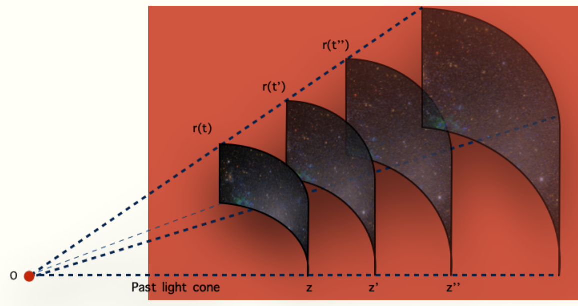
DES-Y1 DATA





The midpoint & geometric do a great job!

LSST Y10 shows similar results



6. Summary

Summary

- Angular correlations functions are very hard to compute
- Limber is the most widely used approximation
- List of issues: accuracy and validity of approximations
- Need for unequal-time correlators and all-angle computations
- Unequal-time EFT does not improve the prediction
- Midpoint approximation better on non-linear scales

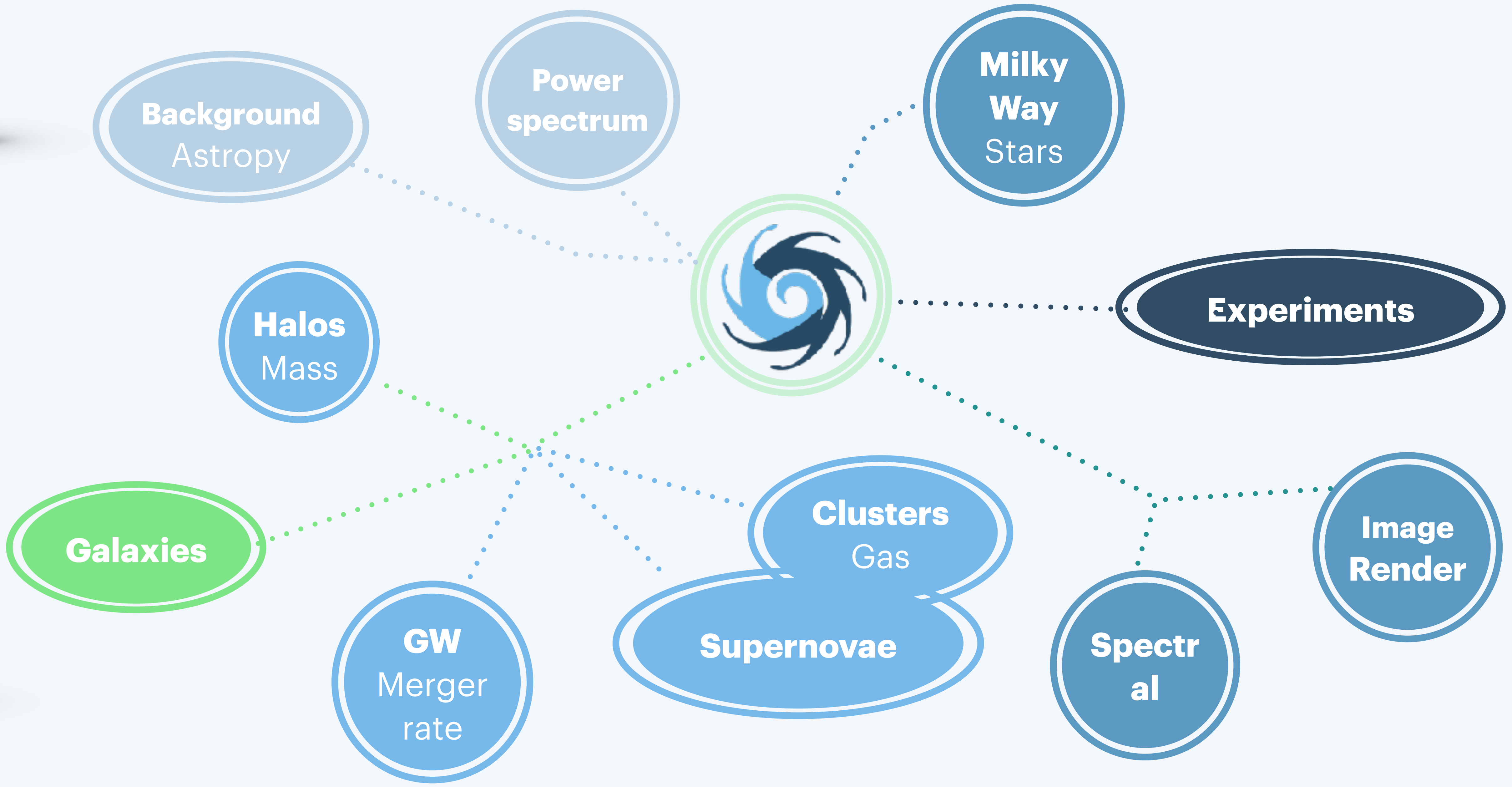
Coming next!

- **Numeric paper (in prep)**
  - All angular scale computations
  - Equal and unequal-time correlators
  - Phyton package **CORFU**
- **Science paper (arXiv 2011.06185)**
  - Unequal-time matter power spectrum at one-loop
  - Phyton package **UNEQUALPY**
  - Analysis of all approximations and validity regimes
  - **Midpoint and Geometric** best to mimic unequal-time features!
  - **Beyond Limber** relevant for **galaxy clustering** and **galaxy-galaxy lensing**.

SkyPy v0.4

Legacy project

- Open-source off-project
- High-quality **python** package
- Aims to be **Astropy** affiliated
- Functionality for **end-to-end simulations**
- Enable **Forward Modelling** & **Machine Learning**

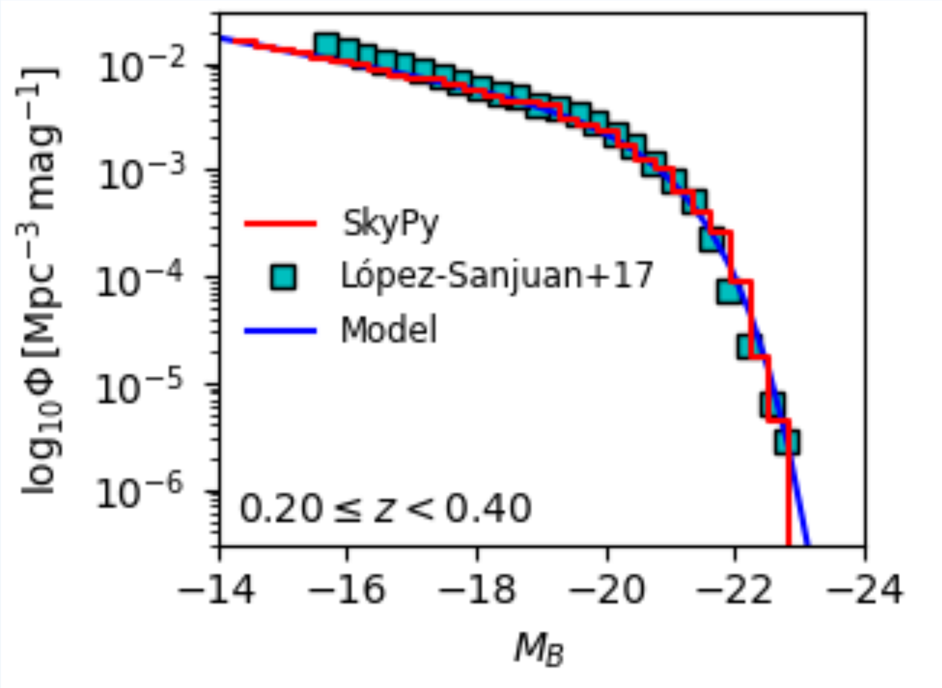


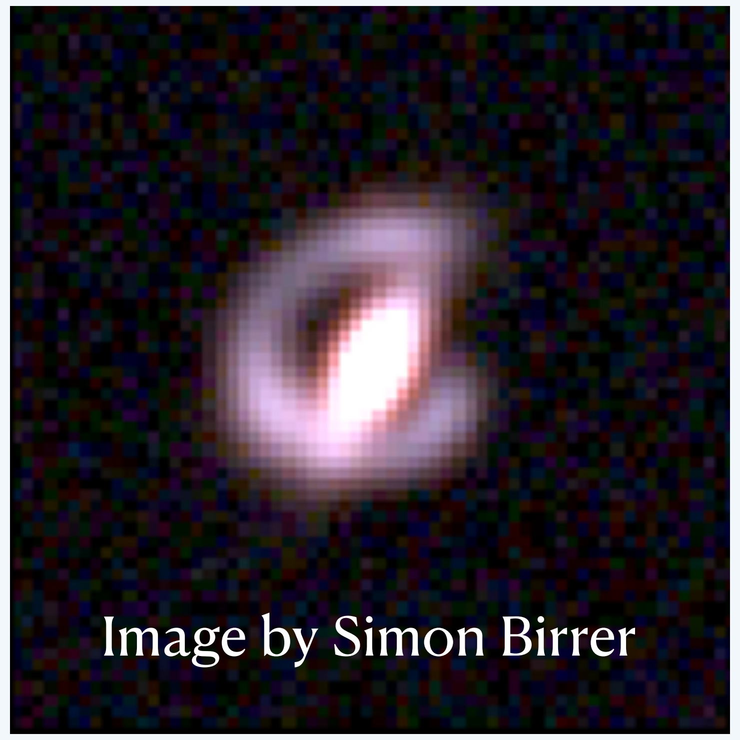
- **GitHub** organisation
- **Issues, pull requests**
- Unit tests, documentation
- **Code review**
- **Infrastructure team**

Key

The **SkyPy Driver** runs end-to-end **pipelines** of functions with **dependencies** to generate outputs.

SkyPy is driven by science projects





Next

- **v0.5** Halo modules.
- **Equality, Diversity and Inclusion**

```
pip install skypy or
conda install -c conda-forge skypy or
git clone https://github.com/skypyproject/skypy.git
import skypy
```


Lucia F. de la Bella

And the SkyPy Collaboration

QUESTIONS?